



Whole-body Compliant Dynamical Contacts in Cognitive Humanoids

## Whole-body Compliant Dynamical Contacts in Cognitive Humanoids proj. no. 600716

**WP1: Systems Integration, Standardization and  
Evaluation on the iCub robot**

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# Outline

- \* T4.1 Improved Models from Real-Time Regression with Latent Contact Type Inference.
  - Parametric identification with arbitrary choices of the base link for accurate and computationally efficient torque and external-force estimation.
  - Maximum-a-posteriori dynamics

# Whole-body dynamics identification for enhanced torque control

## Premises

- \* On the iCub joint torques AND external-forces are estimated from embedded force/torque sensors.
- \* The algorithm is consists of reordering the recursive Newton-Euler algorithm. Specifically, the base link is chosen to coincide with the perceived location of external forces (requires skin sensor).

## Problem

- \* Improving the above procedure by identifying the dynamic model.

# Whole-body dynamics identification for enhanced torque control

## Previous results

- \* Parameters estimated from base-force can be used to compute joint-torques.
  - \* With base-force sensing the so called base parameters can be identified.
  - \* With joint-torque sensing only a subset of the base parameters can be identified (Ayusawa et al., 2013).

## Problem (reformulated)

- \* Parameters estimated from base-force can be used to compute joint-torques AND external forces?

# Identification with torque@joints

$$\begin{bmatrix} \mathbf{I}^c & \mathbf{F} \\ \mathbf{F}^\top & \mathbf{H} \end{bmatrix} \begin{bmatrix} {}^b \mathbf{a}_b \\ \ddot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{p}^c \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau} \end{bmatrix} + \sum_{l \in L} \begin{bmatrix} {}^b \mathbf{X}_l^* \\ \mathbf{J}_l^\top \end{bmatrix} \mathbf{f}_l$$

Articulated rigid body dynamics

$$\begin{bmatrix} \mathbf{I}^c & \mathbf{F} \\ \mathbf{F}^\top & \mathbf{H} \end{bmatrix} \begin{bmatrix} {}^b \mathbf{a}_b \\ \ddot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{p}^c \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau} \end{bmatrix} + \sum_{l \in L} \begin{bmatrix} {}^b \mathbf{X}_l^* \\ \mathbf{J}_l^\top \end{bmatrix} \mathbf{f}_l$$

Joint space dynamics

$$\boldsymbol{\tau} = \mathbf{F}^\top \mathbf{a}_b + \mathbf{H}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}}) - \sum_{l \in L} \mathbf{J}_l^\top \mathbf{f}_l$$

Assumption: known forces and base accelerations

$$\boldsymbol{\tau} = \mathbf{F}^\top \mathbf{a}_b + \mathbf{Y}_\tau(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \boldsymbol{\phi} - \sum_{l \in L} \mathbf{J}_l^\top \mathbf{f}_l$$

$$\boldsymbol{\phi}_l^\top = [m_l \quad m_l \mathbf{c}_l^\top \quad \text{vech}(\bar{\mathbf{I}}_l)^\top]^\top \in \mathbb{R}^{10},$$

Parametric representation

$$\boldsymbol{\tau} = \mathbf{F}^\top \mathbf{a}_b + \mathbf{Y}_\tau(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \mathbf{B}_\tau \boldsymbol{\phi}_\tau - \sum_{l \in L} \mathbf{J}_l^\top \mathbf{f}_l$$

Identifiable parameters

# Identification with force@base

$$\begin{bmatrix} \mathbf{I}^c & \mathbf{F} \\ \mathbf{F}^\top & \mathbf{H} \end{bmatrix} \begin{bmatrix} {}^b \mathbf{a}_b \\ \ddot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{p}^c \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau} \end{bmatrix} + \sum_{l \in L} \begin{bmatrix} {}^b \mathbf{X}_l^* \\ \mathbf{J}_l^\top \end{bmatrix} \mathbf{f}_l$$

Articulated rigid body dynamics

$$\begin{bmatrix} \mathbf{I}^c & \mathbf{F} \\ \mathbf{F}^\top & \mathbf{H} \end{bmatrix} \begin{bmatrix} {}^b \mathbf{a}_b \\ \ddot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{p}^c \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau} \end{bmatrix} + \sum_{l \in L} \begin{bmatrix} {}^b \mathbf{X}_l^* \\ \mathbf{J}_l^\top \end{bmatrix} \mathbf{f}_l$$

Base dynamics

$$\mathbf{I}^c \mathbf{a}_b + \mathbf{F} \ddot{\mathbf{q}} + \mathbf{p}^c = \mathbf{f}_b + \sum_{\substack{l \in L \\ l \neq b}} {}^b \mathbf{X}_l^* \mathbf{f}_l$$

Assumption: known forces

$$\mathbf{Y}_b(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \boldsymbol{\phi} = \mathbf{f}_b + \sum_{\substack{l \in L \\ l \neq b}} {}^b \mathbf{X}_l^* \mathbf{f}_l$$

Parametric representation

$$\mathbf{Y}_b(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \mathbf{B}_b \boldsymbol{\phi}_b = \mathbf{f}_b + \sum_{\substack{l \in L \\ l \neq b}} {}^b \mathbf{X}_l^* \mathbf{f}_l$$

Base parameters

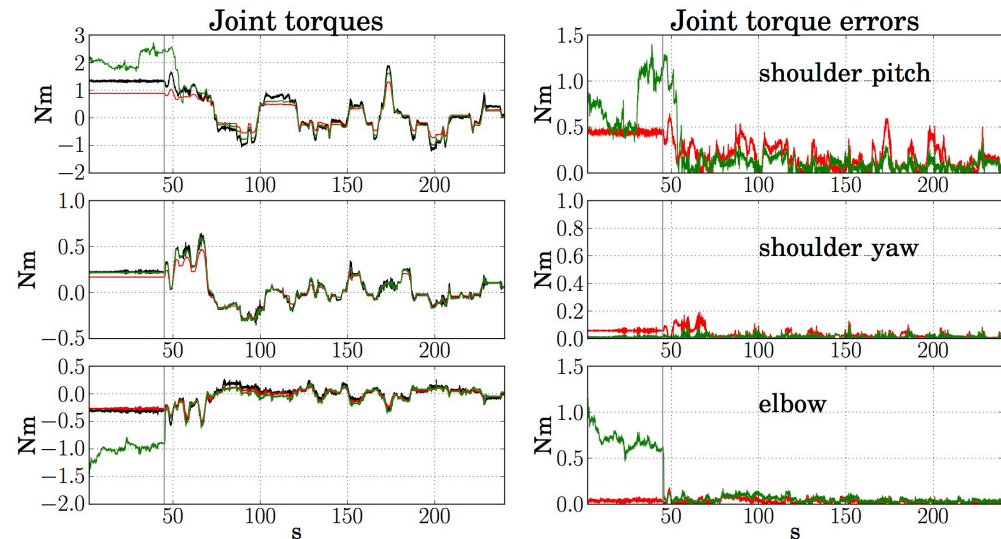
# Identification with force@base

Proposition (Ayusawa et al., 2013)

*The indefinable parameters subspace associated to joint torque measurements is a subspace of the one associated to base force measurements. In other terms, the parameters estimated with base forces can be used to predict joint torques.*

Proposition (Traversaro et al., 2015)

*The indefinable parameters subspace associated with different choices of the base link coincides with the subspace associated to a specific choice of the base link. In other terms, the parameters estimated with base forces can be used to predict joint torques and external-forces.*



Open-source code:  
<https://github.com/robotology-playground/idyntree>  
S. Traversaro et al. (ICRA 2015)

# Maximum-a-posteriori dynamics

## Motivations:

- \* Cons: classical dynamics computations (e.g. inverse dynamics) rely on a subset of available sensors (e.g. inverse dynamics ).
- \* Pros: classical computations are computationally efficient, e.g. RNEA is  $O(n)$ .

## Problem:

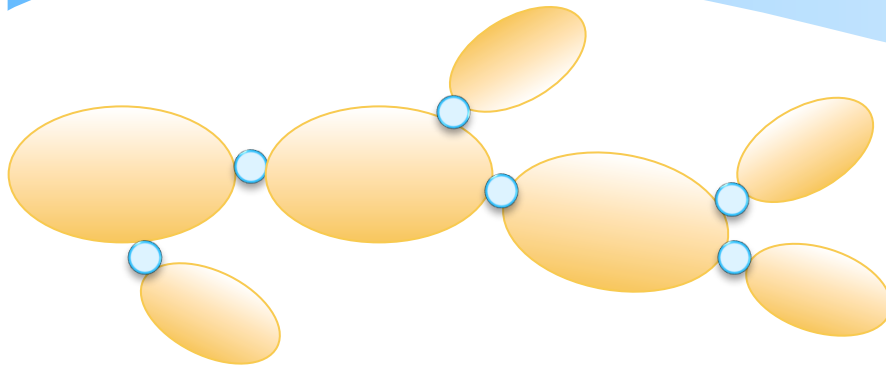
- \* Efficiently compute the dynamics by exploiting all available sensors, including force/torque sensors, gyros and accelerometers.



# Outline

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# Notation



$N_B$ : number of links

Joint quantities

$q_i$ : the joint  $i$  position,  
 $\dot{q}_i$ : the joint  $i$  velocity,  
 $\ddot{q}_i$ : the joint  $i$  acceleration,  
 $\tau_i$ : the joint  $i$  torque.

Link quantities

$v_i$ : the link spatial velocity,  
 $a_i$ : the link spatial accelerations,  
 $f_i$ : the spatial force transmitted to body  $i$  from  $\lambda(i)$ ,  
 $f_i^x$ : external forces acting on body  $i$ .

Dynamic variables

$$d_i = \begin{bmatrix} a_i \\ f_i \\ \tau_i \\ f_i^x \\ \ddot{q}_i \end{bmatrix}, \quad d = \begin{bmatrix} d_1 \\ \vdots \\ d_{N_B} \end{bmatrix}$$

Joint positions

$$q = \begin{bmatrix} q_1 \\ \vdots \\ q_{N_B} \end{bmatrix}$$

Joint velocities

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_{N_B} \end{bmatrix}$$

# Spatial transformations

## Spatial transformations

${}^j \mathbf{X}_i$ : motion-vector transform from link  $i$  to  $j$ ,

${}^j \mathbf{X}_i^*$ : force-vector transform from link  $i$  to  $j$ ,

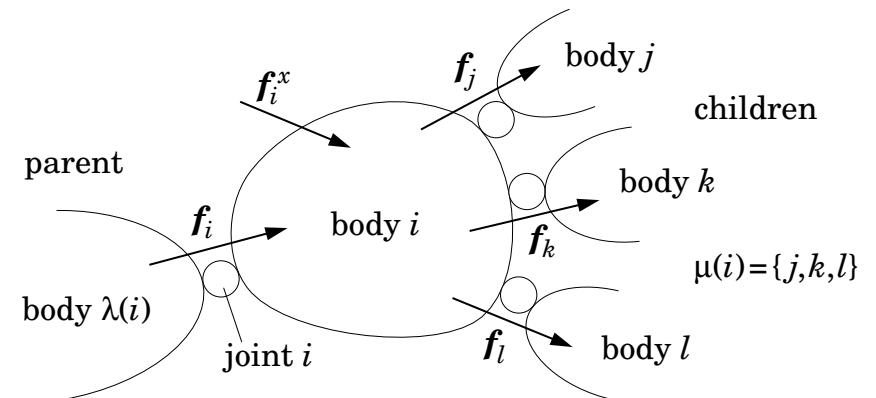
$\mathbf{I}_i$ : spatial inertia tensor link  $i$ ,

$(\mathbf{v})_l$ : linear component,

$(\mathbf{v})_a$ : angular component,

$\times$ : cross product on spatial motions,

$\times^*$ : cross product on spatial forces.



## Link velocities

$$\mathbf{v}_i = {}^i \mathbf{X}_{\lambda(i)} \mathbf{v}_{\lambda(i)} + \mathbf{S}_i \dot{\mathbf{q}}_i,$$



$$\mathbf{v}_i(\mathbf{q}, \dot{\mathbf{q}})$$

# Measurement equation and dynamical consistency

Dynamic and kinematic constraints

$$\left. \begin{aligned} \boldsymbol{\tau}_i &= \mathbf{S}_i^\top \mathbf{f}_i, \\ \mathbf{a}_i &= {}^i\mathbf{X}_{\lambda(i)}(\mathbf{q}_i)\mathbf{a}_{\lambda(i)} + \mathbf{S}_i\ddot{\mathbf{q}}_i + \mathbf{v}_i \times \mathbf{S}_i\dot{\mathbf{q}}_i, \\ \mathbf{f}_i &= \mathbf{I}_i\mathbf{a}_i - \mathbf{f}_i^x + \sum_{j \in \mu(i)} {}^i\mathbf{X}_j^*(\mathbf{q}_j)\mathbf{f}_j + \mathbf{v}_i \times^* \mathbf{I}_i\mathbf{v}_i \end{aligned} \right\}$$

$$\mathbf{D}(\mathbf{q})\mathbf{d} + \mathbf{b}_D(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}$$

Measurement equations

$$\left. \begin{aligned} \mathbf{y}_{acc} &= ({}^s\mathbf{X}_i\mathbf{a}_i)_l + ({}^s\mathbf{X}_i\mathbf{v}_i)_a \times ({}^s\mathbf{X}_i\mathbf{v}_i)_l, \\ \mathbf{y}_{gyr} &= {}^sR_i(\mathbf{v}_i)_a, \\ \mathbf{y}_{fts} &= {}^s\mathbf{X}_i^*(\mathbf{f}_i - \mathbf{I}_{im}\mathbf{a}_i - \mathbf{v}_i \times^* \mathbf{I}_{im}\mathbf{v}_i), \\ \mathbf{y}_{skn} &= {}^s\mathbf{X}_i^* \mathbf{P}_\perp^s \mathbf{f}_i^x. \end{aligned} \right\}$$

$$\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{d} + \mathbf{b}_Y(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{y}.$$

$$\begin{bmatrix} \mathbf{D}(\mathbf{q}) \\ \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} \mathbf{d} + \begin{bmatrix} \mathbf{b}_D(\mathbf{q}, \dot{\mathbf{q}}) \\ \mathbf{b}_Y(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{y} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{D}(\mathbf{q}) \\ \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} \text{ will be assumed full column rank,}$$

# Problem 1: estimation

ESTIMATION: estimate  $\mathbf{d}$  given  $\mathbf{y}$ .

$$p(\mathbf{d}, \mathbf{y}) = p(\mathbf{d})p(\mathbf{y}|\mathbf{d})$$

$$\mathbf{D}(\mathbf{q})\mathbf{d} + \mathbf{b}_D(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}$$

$$\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{d} + \mathbf{b}_Y(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{y}.$$

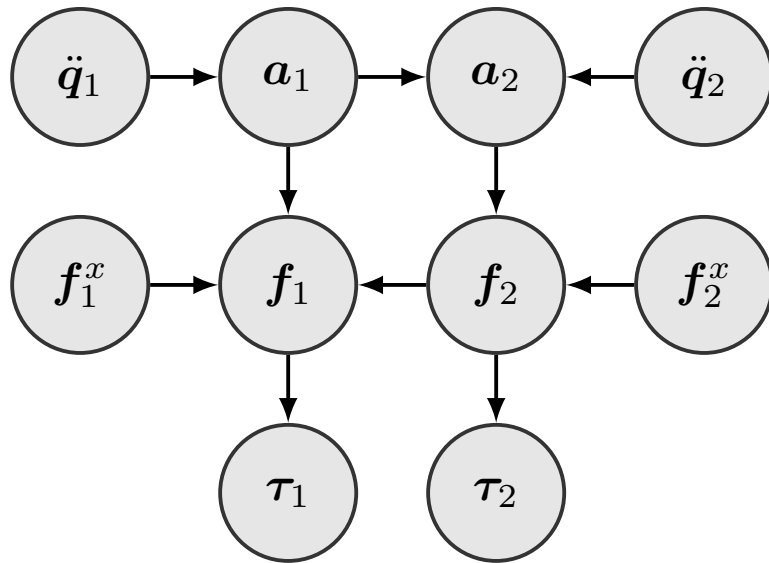
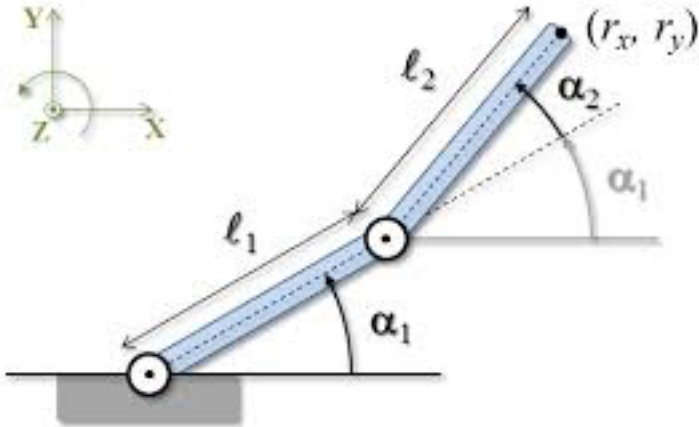
$$p(\mathbf{d}) \propto \exp \{ \mathbf{e}(\mathbf{d})^\top \boldsymbol{\Sigma}_D^{-1} \mathbf{e}(\mathbf{d}) \},$$
$$\mathbf{e}(\mathbf{d}) = \mathbf{D}(\mathbf{q})\mathbf{d} + \mathbf{b}_D(\mathbf{q}, \dot{\mathbf{q}}),$$

$$p(\mathbf{y}|\mathbf{d}) \sim \mathcal{N}(\boldsymbol{\mu}_y, \boldsymbol{\Sigma}_y),$$
$$\boldsymbol{\mu}_y = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{d} + \mathbf{b}_Y(\mathbf{q}, \dot{\mathbf{q}}),$$

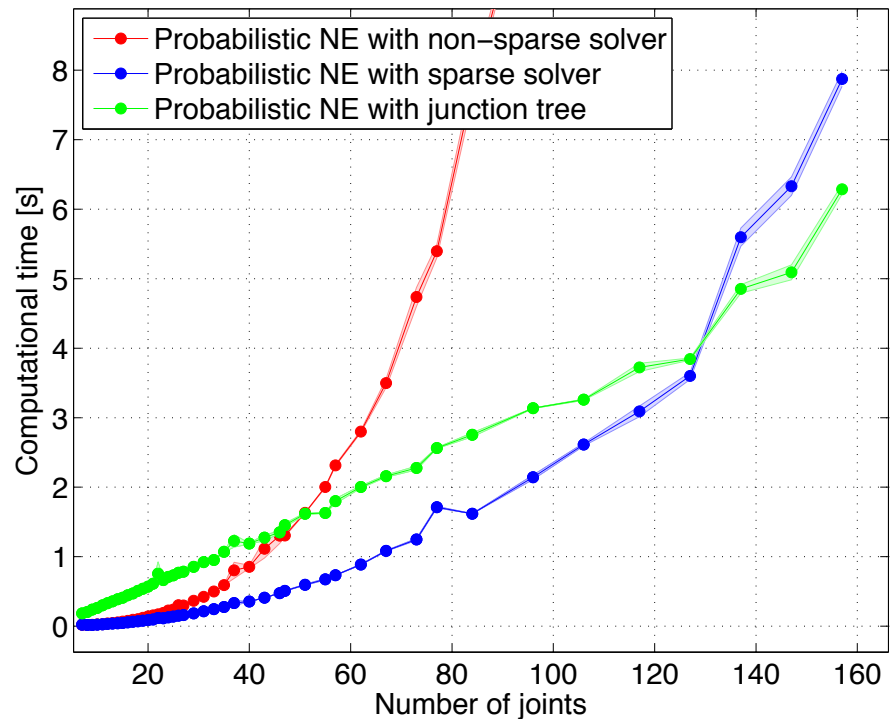
$$\mathbf{d}_{map} = \arg \max_{\mathbf{d}} p(\mathbf{d}|\mathbf{y}),$$

$$\mathbf{d}_{map} = (\mathbf{D}^\top \boldsymbol{\Sigma}_D^{-1} \mathbf{D} + \mathbf{Y}^\top \boldsymbol{\Sigma}_y^{-1} \mathbf{Y})^{-1} \cdot (\mathbf{Y}^\top \boldsymbol{\Sigma}_y^{-1} (\mathbf{y} - \mathbf{b}_Y) - \mathbf{D}^\top \boldsymbol{\Sigma}_D^{-1} \mathbf{b}_D).$$

# Computational efficiency



Computations have been optimized by exploiting the matrices sparsity.



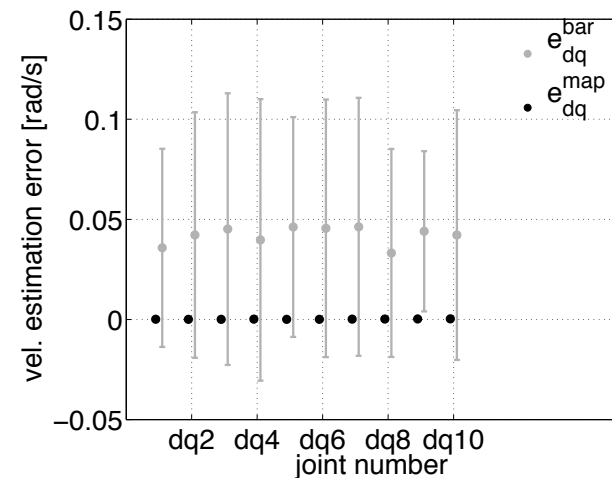
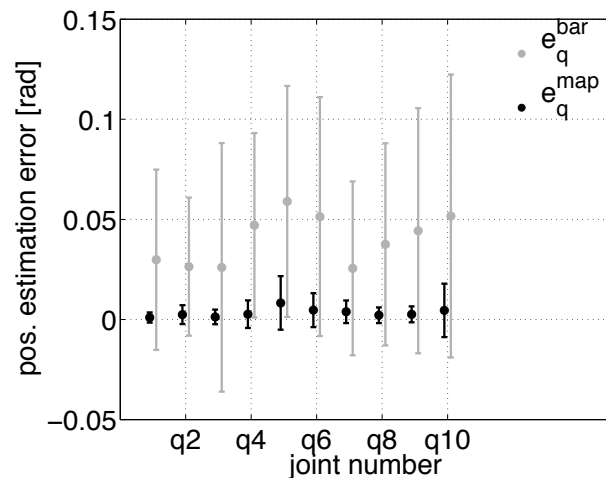
# Problem 2: state estimation

STATE ESTIMATION: estimate  $q, \dot{q}$  given  $y$ .

$$\begin{bmatrix} D(q) \\ Y(q, \dot{q}) \end{bmatrix} d + \begin{bmatrix} b_D(q, \dot{q}) \\ b_Y(q, \dot{q}) \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix}, \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

First order  
approximation  
around  $\bar{d}, \bar{x}$

$$\begin{bmatrix} D(\bar{x}) & \partial b_D(\bar{d}, \bar{x}) \\ Y(\bar{x}) & \partial b_Y(\bar{d}, \bar{x}) \end{bmatrix} \begin{bmatrix} d \\ x \end{bmatrix} + \begin{bmatrix} b_D(\bar{x}) - \partial b_D(\bar{d}, \bar{x})\bar{x} \\ b_Y(\bar{x}) - \partial b_Y(\bar{d}, \bar{x})\bar{x} \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix}$$

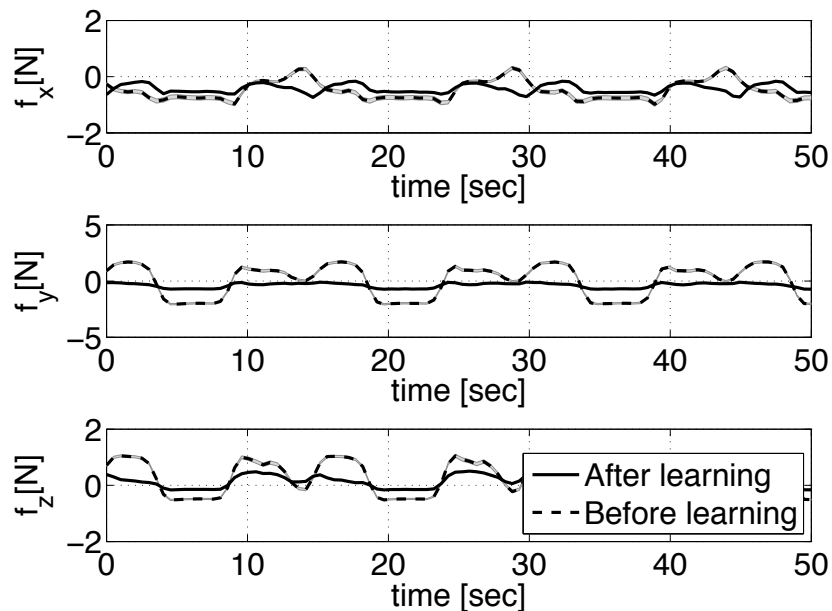


# Problem 3: hyper-parameter estimation

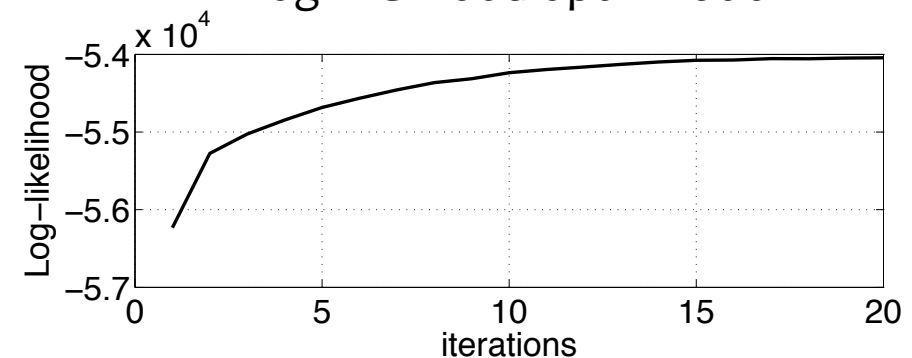
**HYPERPARAMETER ESTIMATION: estimate  $\Sigma_y$  and  $\Sigma_d$  given  $y$ .**

$$\begin{aligned} \text{E-step} \quad & Q(\theta|\theta^t) = \sum_t E_{d|y^t; \theta^t} [\log p(d, y^t; \theta, t)], \\ \text{M-step} \quad & \theta^{t+1} = \arg \max_{\theta} Q(\theta|\theta^t). \end{aligned}$$

External force estimation



Log-likelihood optimization



Open-source code: [https://github.com/iron76/bnt\\_time\\_varying](https://github.com/iron76/bnt_time_varying)