ISTITUTO ITALIANO DI TECNOLOGIA

# Motion-Force Control of Humanoid Robots

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# Task Space Inverse Dynamics

A(nother)
control framework
for **prioritized**
motion and **force** control

# System Dynamics

| | Unconstrained | Constrained |
|---|---|---|
| Fully actuated | $M\ddot{q} + h = \tau$ | $M\ddot{q} + h - J_c^T f = \tau$<br>$J_c\ddot{q} = -\dot{J}_c\dot{q}$ |
| Under actuated | $M\ddot{q} + h = S^T\tau$ | $M\ddot{q} + h - J_c^T f = S^T\tau$<br>$J_c\ddot{q} = -\dot{J}_c\dot{q}$ |

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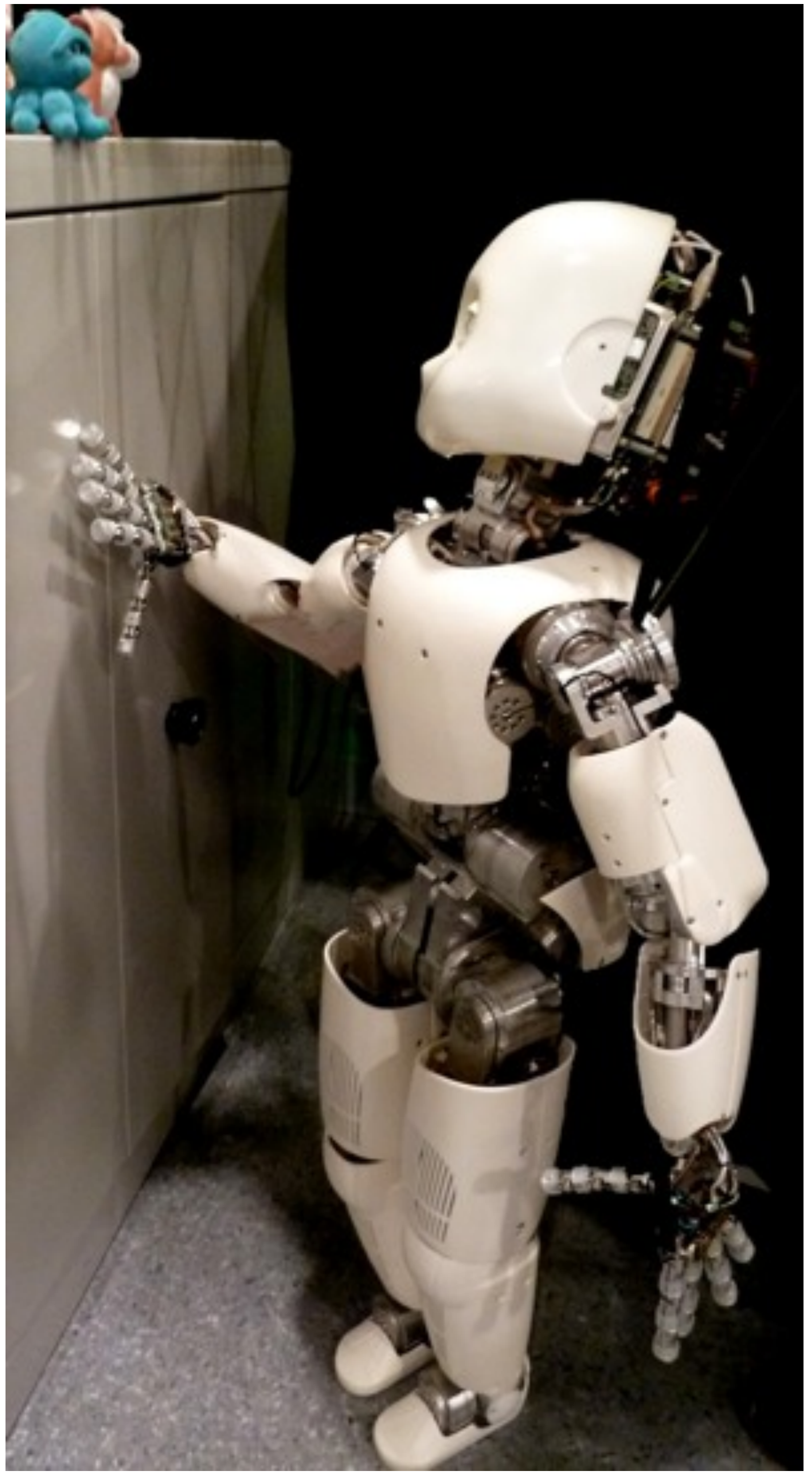
# Prioritized Control Frameworks

- Efficiency
- Capabilities
  - force control
  - underactuation
  - inequalities
- Optimality

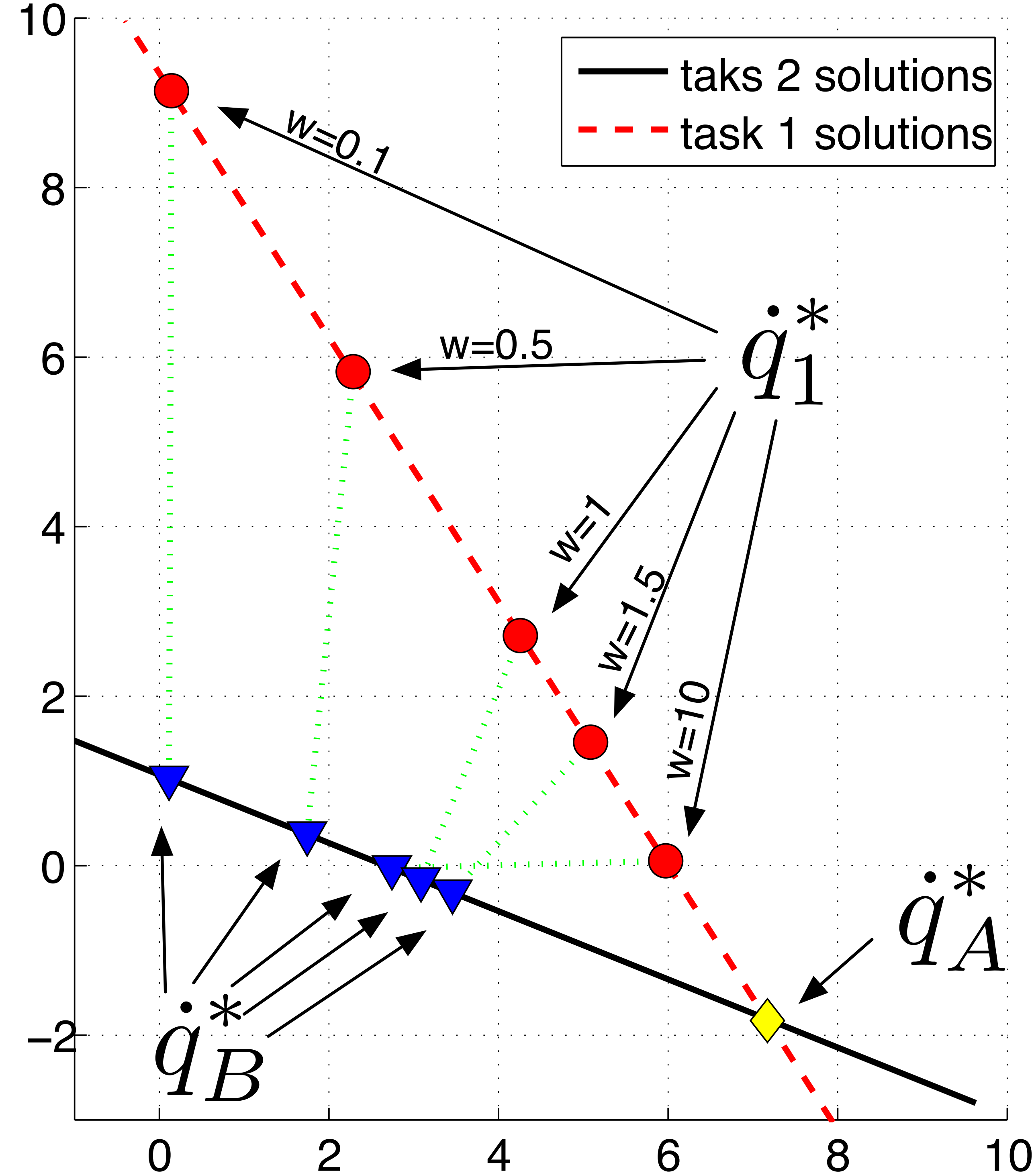


# Optimality

Two velocity tasks:

$$\dot{x}_2 = J_2 \dot{q}$$ — *Higher priority*

$$\dot{x}_1 = J_1 \dot{q}$$ — *Lower priority*




Two approaches:

$$\dot{q}_A^* = J_2^+ \dot{x}_2^* + (J_1 N_2)^+ (\dot{x}_1^* - J_1 J_2^+ \dot{x}_2^*)$$

$$\dot{q}_B^* = J_2^+ \dot{x}_2^* + N_2 J_1^+ \dot{x}_1^*$$

# State of the Art

| Framework | Optimal | Efficient | Force Ctrl | Under. | Out. |
|---|---|---|---|---|---|
| TASK SPACE INVERSE DYNAMICS (TSID) | × | × | × | | $\tau$ |
| Peters et al. [2007] (UF) | | × | × | | $\tau$ |
| Sentis and Khatib [2005] (WBCF) | × | | × | (×) | $\tau$ |
| Mistry and Righetti [2011] | | | × | × | $\tau$ |
| Saab et al. [2011a] | × | | × | × | $\tau$ |
| De Lasa and Hertzmann [2009] | × | | × | × | $\tau$ |
| Jeong [2009] | × | × | | | $\tau/\ddot{q}$ |
| Chiaverini [1997] | | × | | | $\dot{q}$ |
| Siciliano and Slotine [1991] | × | × | | | $\dot{q}$ |
| Baerlocher and Boulic [1998] | × | × | | | $\dot{q}$ |
| Nakamura et al. [1987] | | × | | | $\dot{q}/\ddot{q}$ |

# Prioritized Control Problem

Task i Jacobian

Desired acceleration

Cost of task i

$$g_i(\tau) = ||J_i \ddot{q} + \dot{J}_i \dot{q} - \ddot{x}_i^*||^2$$

$$g_i(\tau) = ||f_i - f_i^*||^2$$

Desired force

**Contact constraints**

$$J_c \ddot{q} = -\dot{J}_c \dot{q}$$

$$r_N = \min_{\tau \in \mathbb{R}^n} g_N(\tau) \quad s.t. \quad M\ddot{q} + h = \tau \quad J_c \ddot{q} = -\dot{J}_c \dot{q}$$

$$r_i = \min_{\tau \in \mathbb{R}^n} g_i(\tau) \quad s.t. \quad M\ddot{q} + h = \tau \quad J_c \ddot{q} = -\dot{J}_c \dot{q}$$

$$g_j(\tau) = r_j \quad \forall j > i$$

$$\tau^* = \operatorname*{argmin}_{\tau \in \mathbb{R}^n} ||\ddot{q} - \ddot{q}_0^*|| \quad s.t. \quad M\ddot{q} + h = \tau \quad J_c \ddot{q} = -\dot{J}_c \dot{q}$$

$$g_j(\tau) = r_j \quad \forall j > 0$$

Postural Task

# Prioritized Control Problem

SOLUTION

$$\begin{aligned}
\tau^* &= M\ddot{q}_0^* + h - J_c^T f^* \\
\ddot{q}_i &= \ddot{q}_{i+1} + (J_i N_{p(i)})^+ (\ddot{x}_i^* - \dot{J}_i \dot{q} - J_i \ddot{q}_{i+1}) \\
N_{p(i)} &= N_{p(i+1)} - (J_{i+1} N_{p(i+1)})^+ J_{i+1} N_{p(i+1)}
\end{aligned}$$

[redacted]

| Related to | *Soundness* | | *Optimality* | | *Efficiency* |
| --- | --- | --- | --- | --- | --- |
| Controller | F Error (N) | T2 Error (mm) | T1 Error (mm) | P Error (°) | Computation Time (ms) |
| **TSID** | **0.46** | **0.3** | **0.7** | **10.4** | **0.298** |
| WBCF | 0.46 | 0.1 | 0.9 | 10.4 | 0.681 |
| UF | 0.46 | 139.7 | 145.2 | 6.2 | 0.316 |

- TSID: Task Space Inverse Dynamics
- WBCF: Whole Body Control Framework
- UF: Unifying Framework

# System Dynamics

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| Fully actuated | $M\ddot{q} + h = \tau$ ✓ | $M\ddot{q} + h - J_c^T f = \tau$<br>$J_c\ddot{q} = -\dot{J}_c\dot{q}$ ✓ |
| Under actuated | $M\ddot{q} + h = S^T\tau$ | $M\ddot{q} + h - J_c^T f = S^T\tau$<br>$J_c\ddot{q} = -\dot{J}_c\dot{q}$ |

# Task Space Partial Feedback Linearization

$$\tau^* = N_j^{-1}\ddot{q}_j^* + \bar{S}^T h$$

*Hybrid dynamics*

$$\ddot{q}_j^* = (J\bar{S})_W^+(\ddot{x}^* - \dot{J}\dot{q} + J_b M_b^{-1} h_b) + (I - (J\bar{S})_W^+ J\bar{S})\ddot{q}_{j0}$$

$$J\bar{S} = J_j - J_b M_b^{-1} M_{bj}$$

*Generalized Jacobian*

$$M = \begin{bmatrix} M_b & M_{bj} \\ M_{bj}^T & M_j \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} N_b & N_{bj} \\ N_{bj}^T & N_j \end{bmatrix}$$

[Shkolnik and Tedrake 2008]

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# What would you like to control?

n+6 | n+6-k | k

State = Accelerations + Forces

Control

n

3 CASES

- [some] Motion
- [some] Motion + some Forces
- some Motion + all Forces

# Motion (only) control

$$\tau^* = (N_c S^T)^+ N_c (M\ddot{q}_0 + h)$$

$$\ddot{q}_i = \ddot{q}_{i+1} + (J_i N_{p(i)})^+ (\ddot{x}_i^* - \dot{J}_i \dot{q} - J_i \ddot{q}_{i+1})$$

$$N_{p(i)} = N_{p(i+1)} - (J_{i+1} N_{p(i+1)})^+ J_{i+1} N_{p(i+1)}$$

*No Force Measurements!*

*k≥6* ⇒ *Constraint-consistent motion always feasible* ⇒ *Constraint Nullspace Projection*

[Aghili 2005]

# Motion (only) control

$$\tau^* = (JM_c^{-1}N_cS^T)^+(\ddot{x}^* - \dot{J}\dot{q}$$
$$+ JM_c^{-1}(N_ch + J_c^+\dot{J}_c\dot{q})) + N\tau_0$$

$$M_c = N_cM + J_c^+J_c$$

*k<6* ⇒ *Constraint-consistent motion may be unfeasible*

# Partial Force Control

$$\tau^* = (N_c S^T)^+ N_c (M\ddot{q}_0 + h - J_f^T f^*)$$

$$\ddot{q}_i = \ddot{q}_{i+1} + (J_i N_{p(i)})^+ (\ddot{x}_i^* - \dot{J}_i \dot{q} - J_i \ddot{q}_{i+1})$$

$$N_{p(i)} = N_{p(i+1)} - (J_{i+1} N_{p(i+1)})^+ J_{i+1} N_{p(i+1)}$$

*k≥6* ⇒ *Constraint-consistent motion is always feasible* ⇒ *Constraint Nullspace Projection*

# Partial Force Control

$$\tau^* = (\hat{J}\hat{M}^{-1}V_2^T S^T)^+((J_f J_c^+ \dot{J}_c - \dot{J}_f)\dot{q} + \hat{J}\hat{M}^{-1}\hat{h} - \hat{J}\hat{M}^{-1}\hat{J}^T f^*)$$

$$\hat{M} = V_2^T M V_2$$

$$\hat{h} = V_2^T h - V_2^T M J_c^+ \dot{J}_c \dot{q}$$

$$\hat{J} = J_f V_2$$

$V_2 =$ Orthogonal base of constraint nullspace

*k<6* ⇒ *Constraint-consistent motion may be unfeasible*

# Complete Force Control

$$\tau^* = -(J_c\bar{S})^T f^* + \boxed{N_j^{-1}\ddot{q}_0 + \bar{S}^T h}$$

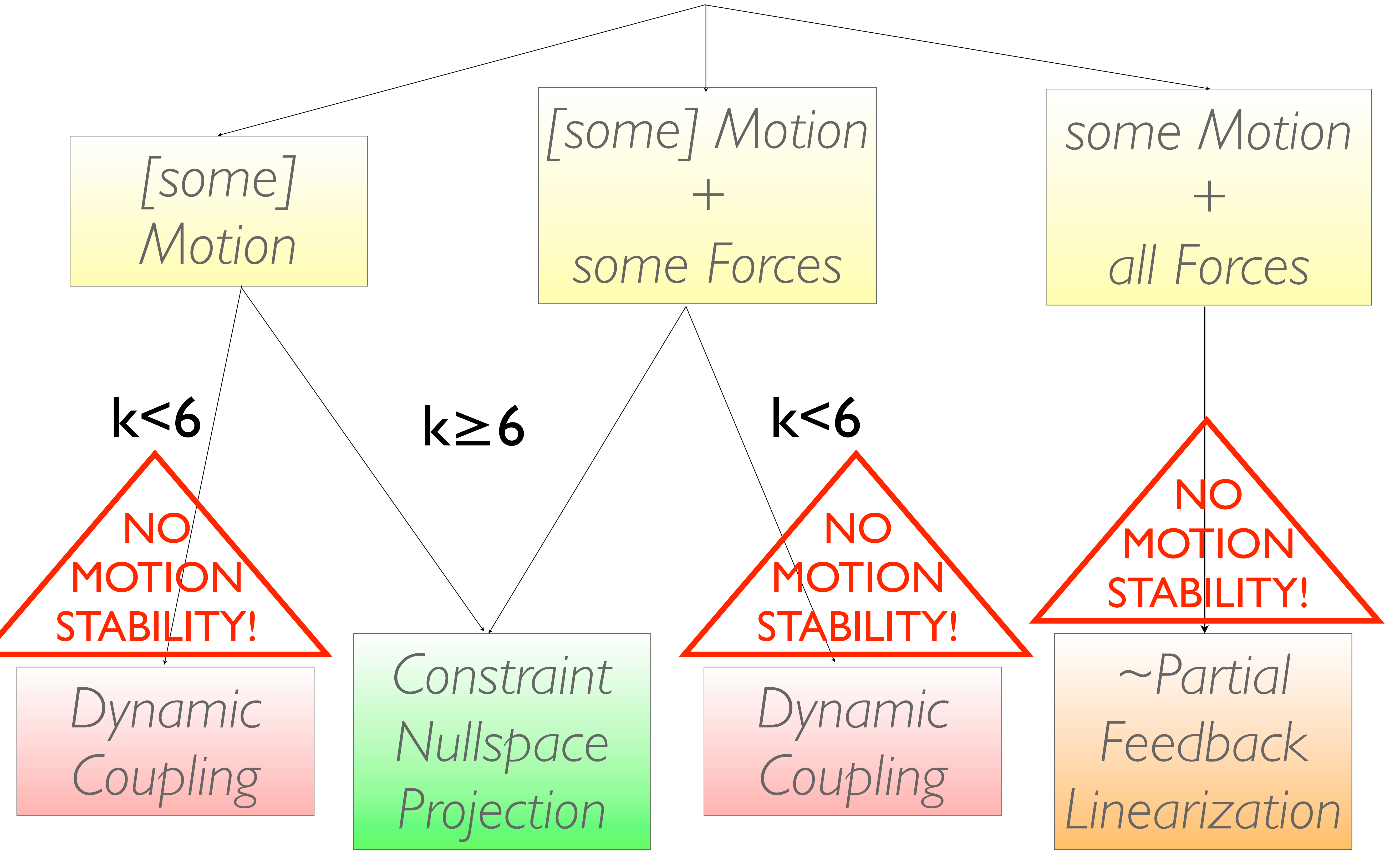
PFL

$$\ddot{q}_i = \ddot{q}_{i+1} + (J_i\bar{S}N_{p(i)})^+(\ddot{x}_i^* - \dot{J}_i\dot{q} + J_i(U^T M_b^{-1}(h_b - J_{cb}^T f) - \bar{S}\ddot{q}_{i+1}))$$

$$N_{p(i)} = N_{p(i+1)} - (J_{i+1}\bar{S}N_{p(i+1)})^+ J_{i+1}\bar{S}N_{p(i+1)}$$

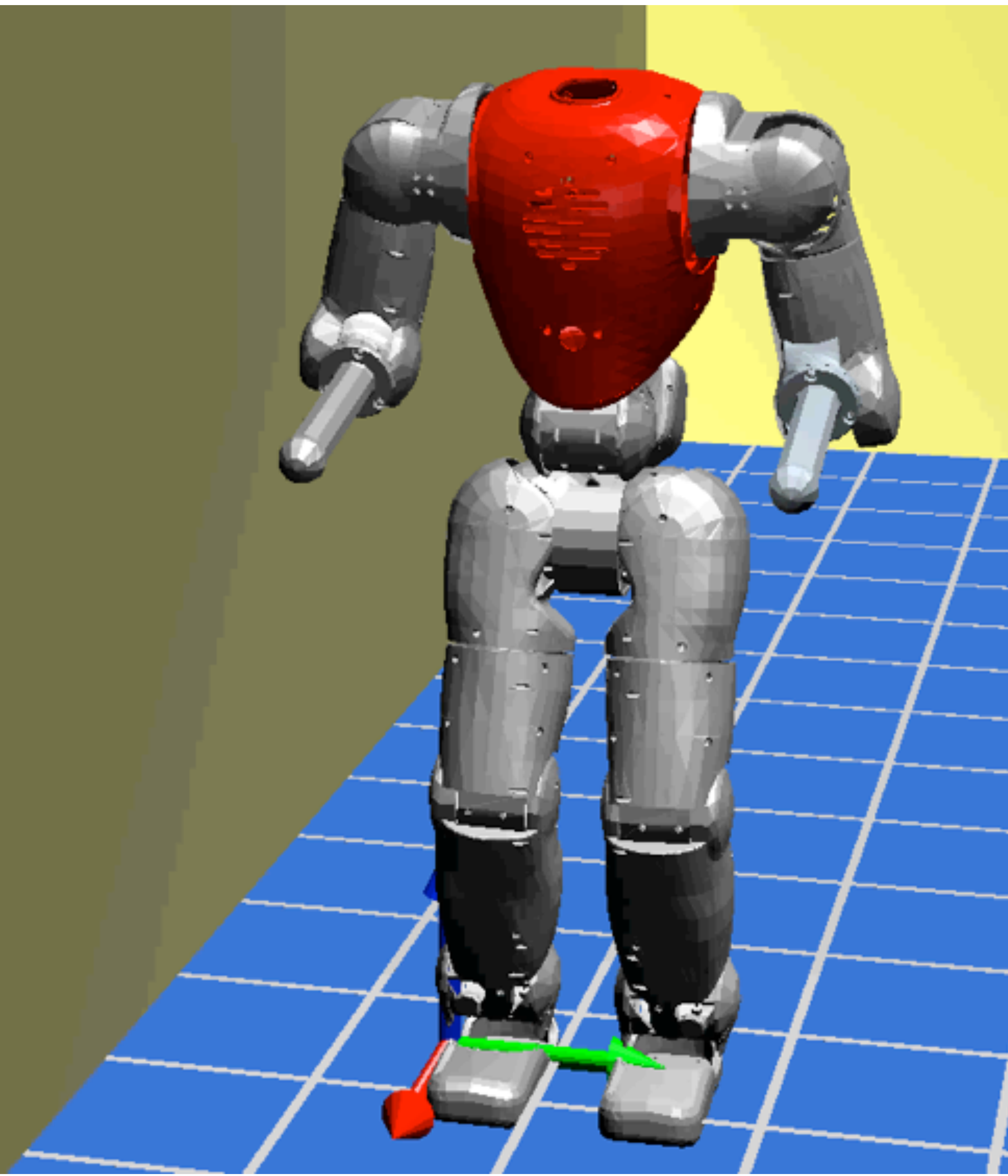
~ *Partial Feedback Linearization* → *Dynamic singularities*

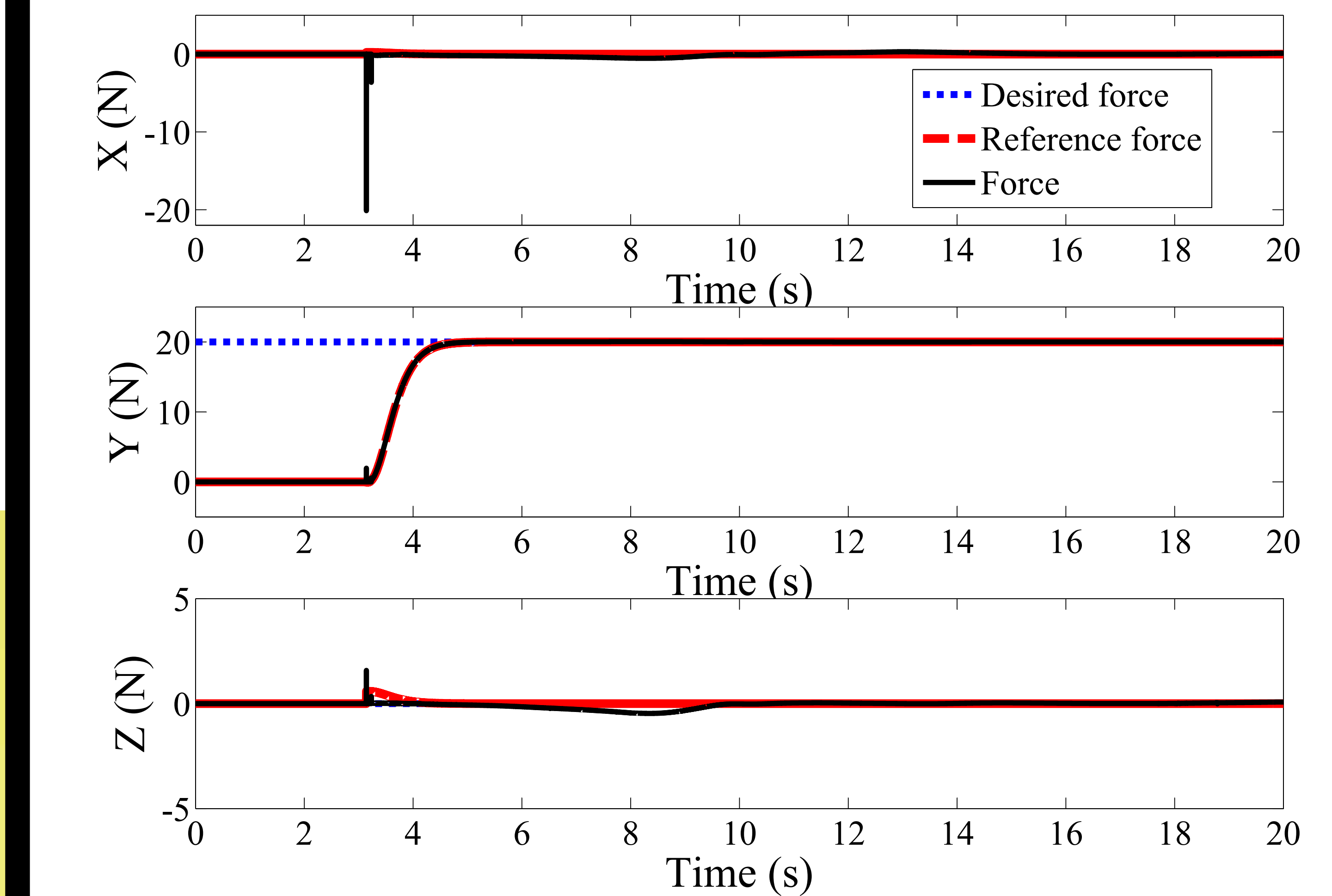
# What would you like to control?

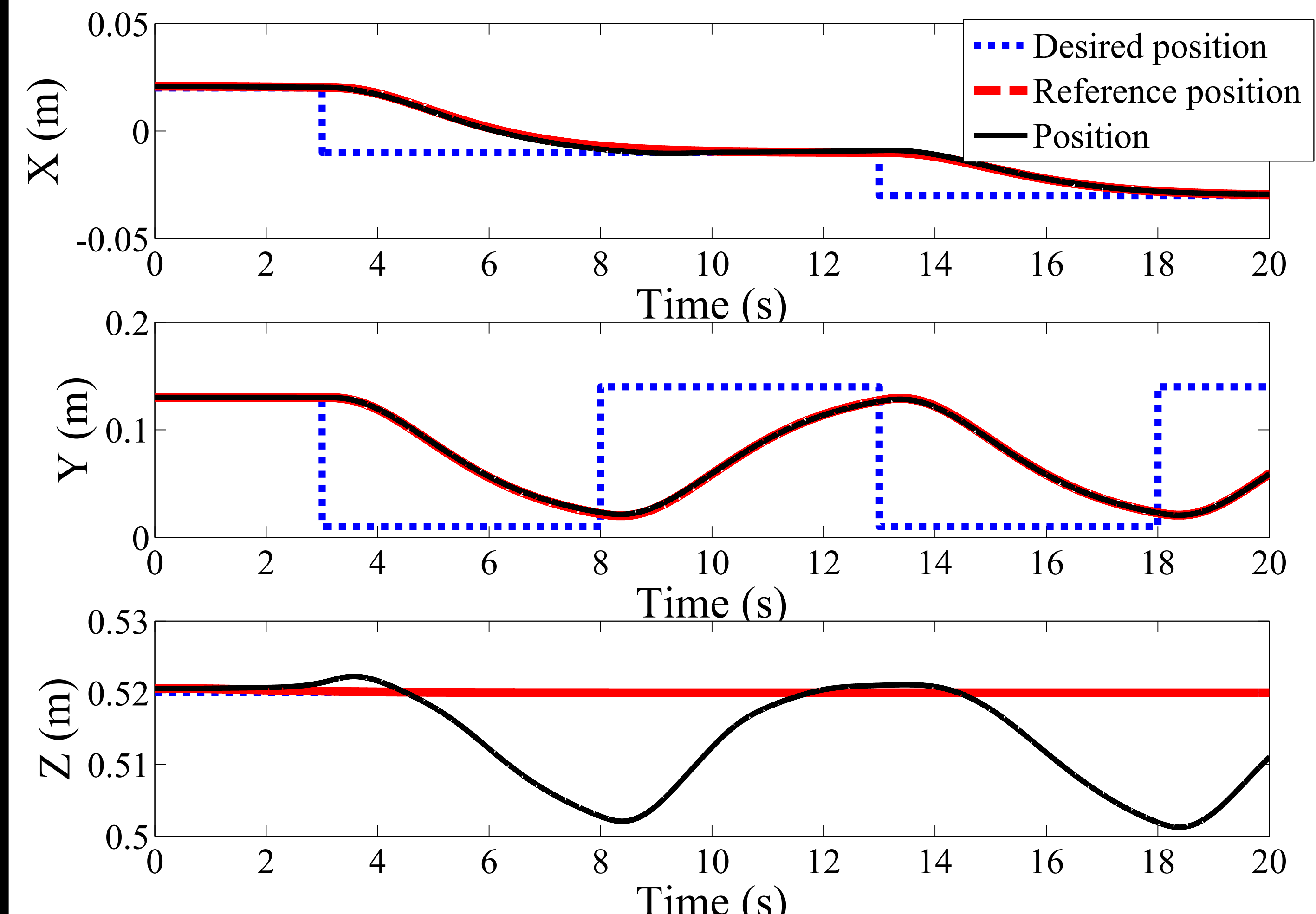



k = number of uncontrolled constraint forces

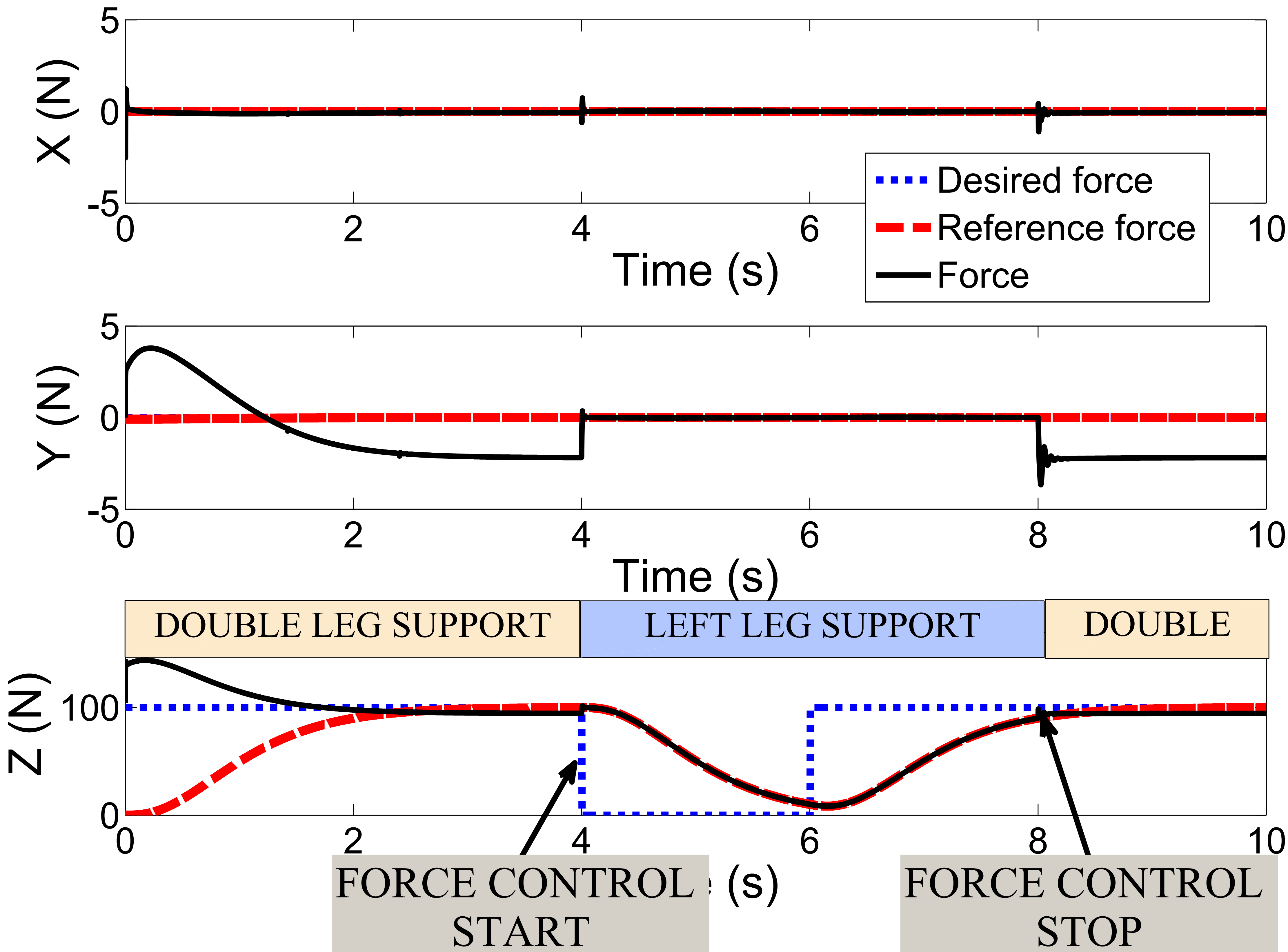
# Partial Force Control





# Switching support leg



# System Dynamics

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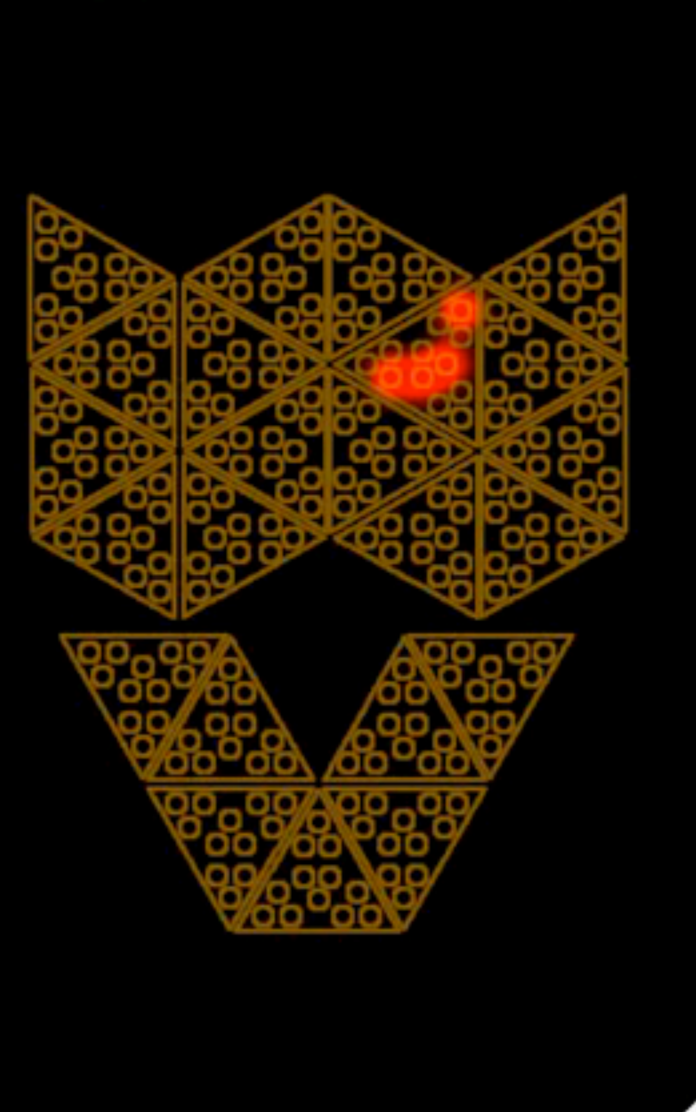
|---|---|---|
| Fully actuated | Inverse dynamics<br>+<br>Prioritized acc. ctrl | Inverse dynamics<br>+<br>Prioritized acc. ctrl |
| Under actuated | P.F.L.<br>+<br>Prioritized acc. ctrl<br>with<br>Generalized Jacobians | $k \geq 6$<br>Constr. Nullspace Proj.<br>~P.F.L.<br>Dynamic coupling<br>$k < 6$ |

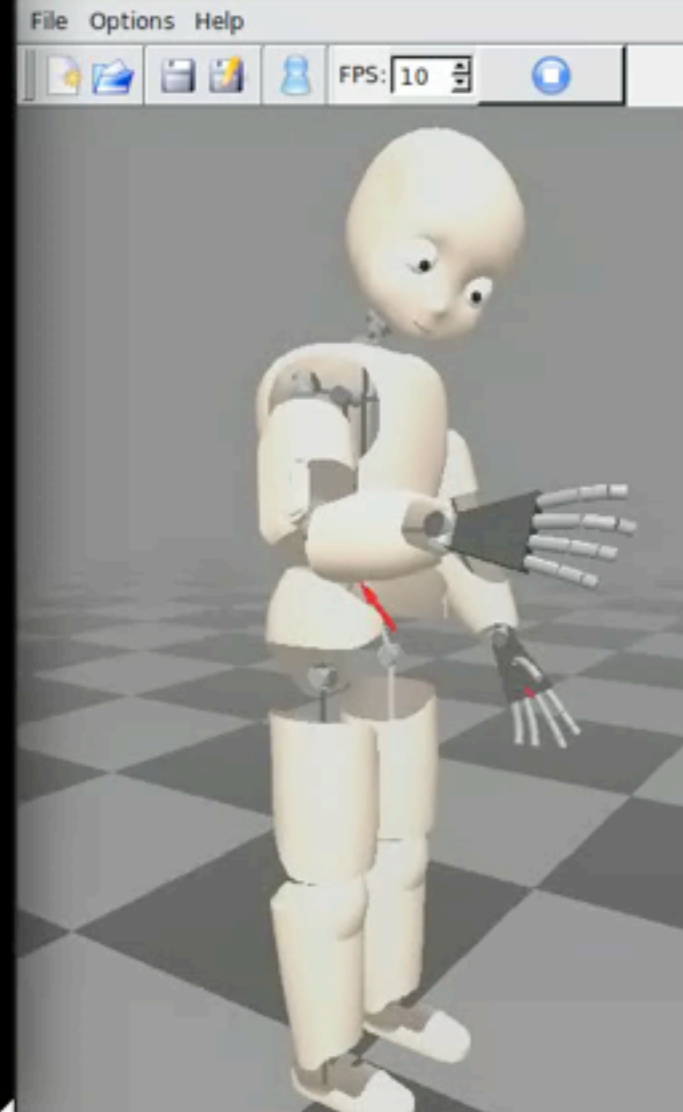
# Future work

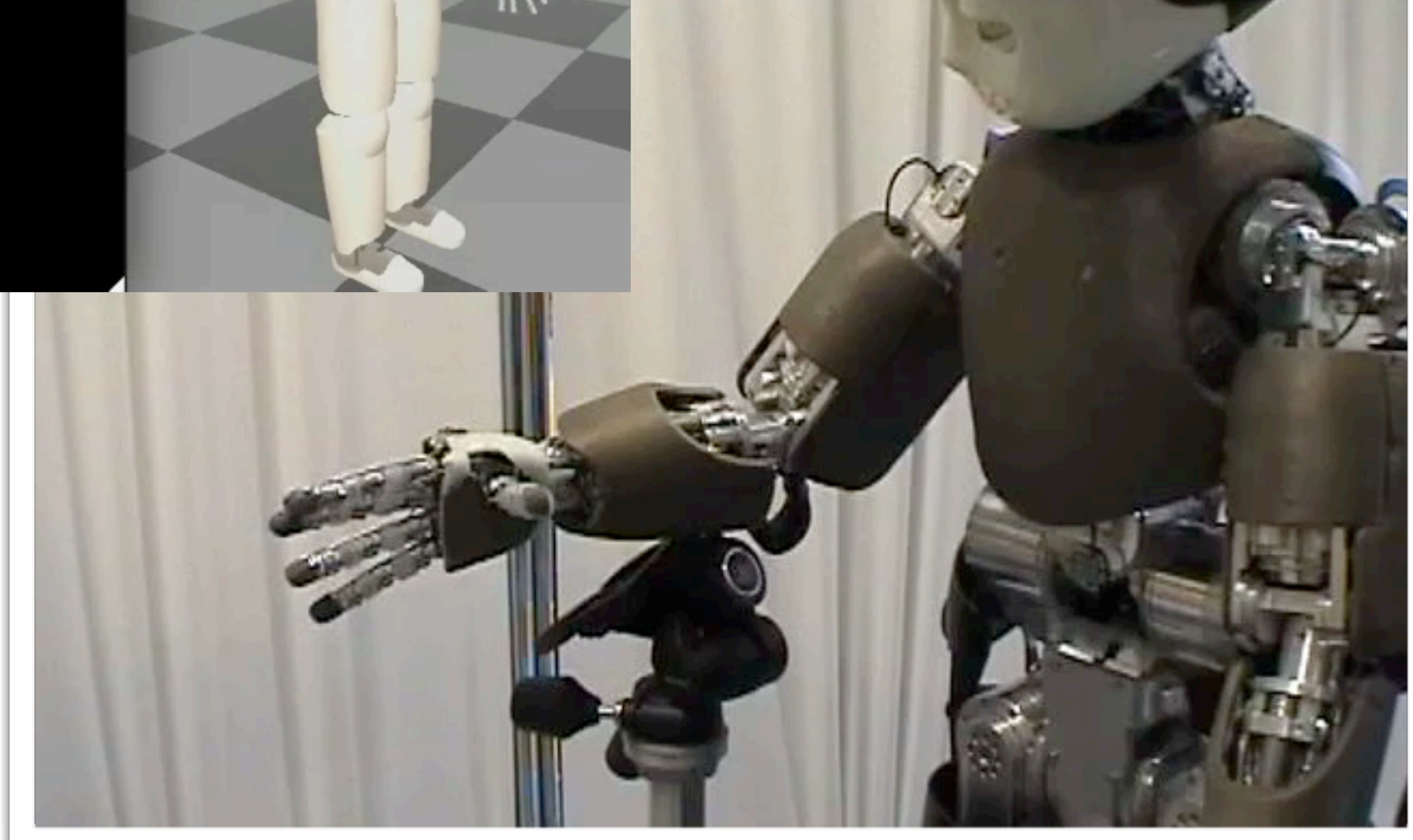
- **inequalities** (joint/torque limits, ZMP, friction)
- **constraint** switching/selection
- instantaneous/local **optimality**
- computational **cost**
- **implementation** (iTaSC, SoT)

Make iCub walk...

# The End





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