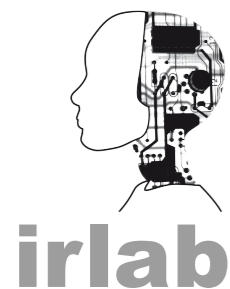


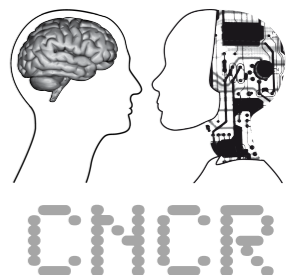
# Exploiting Redundancy to Optimize the Task Space

Michael Mistry

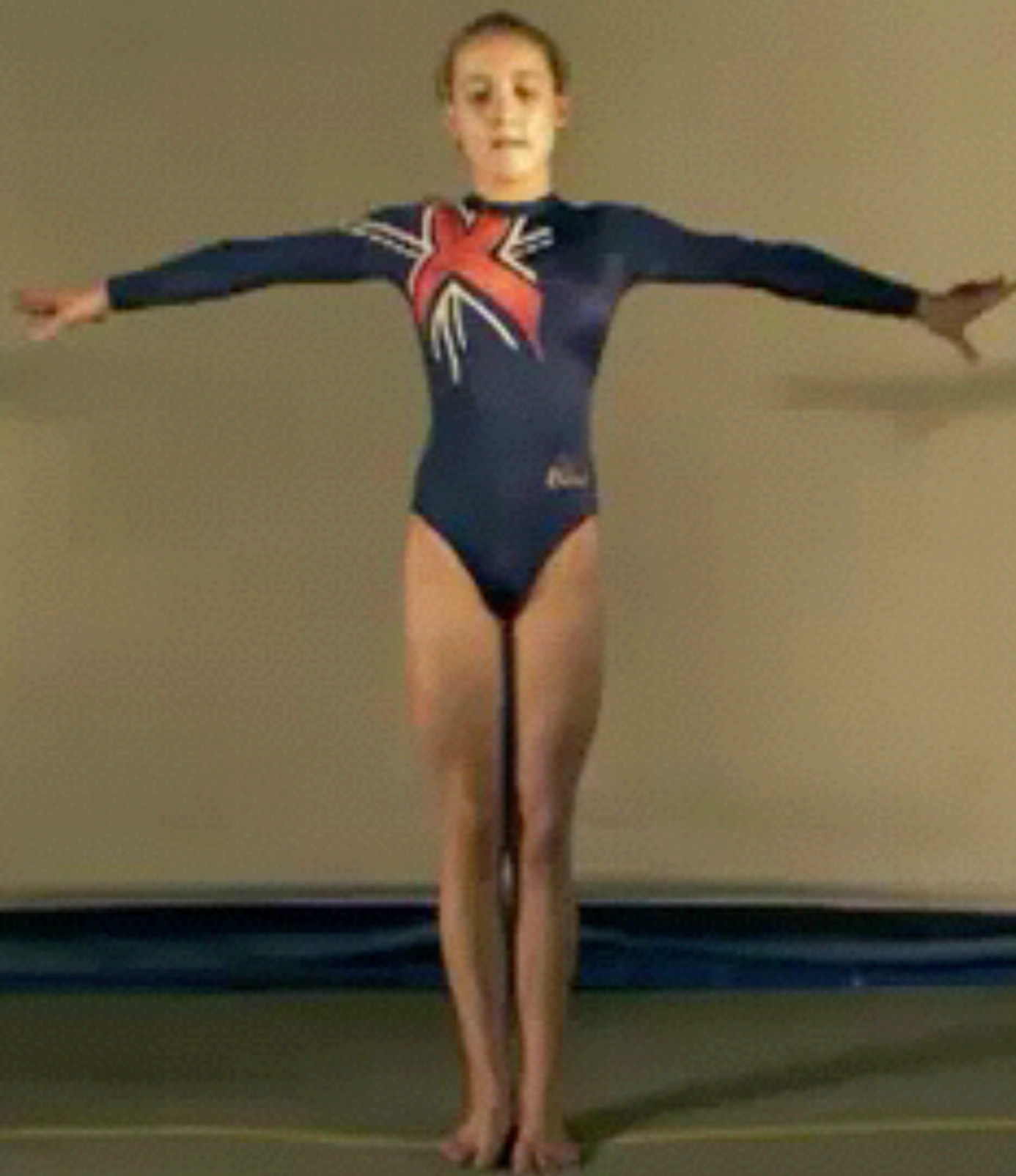
Lecturer in Robotics  
School of Computer Science  
Centre for Computational Neuroscience and Cognitive Robotics



UNIVERSITY OF  
BIRMINGHAM







What is she thinking about?



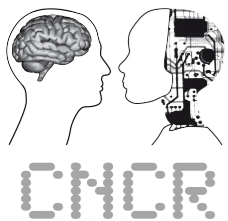
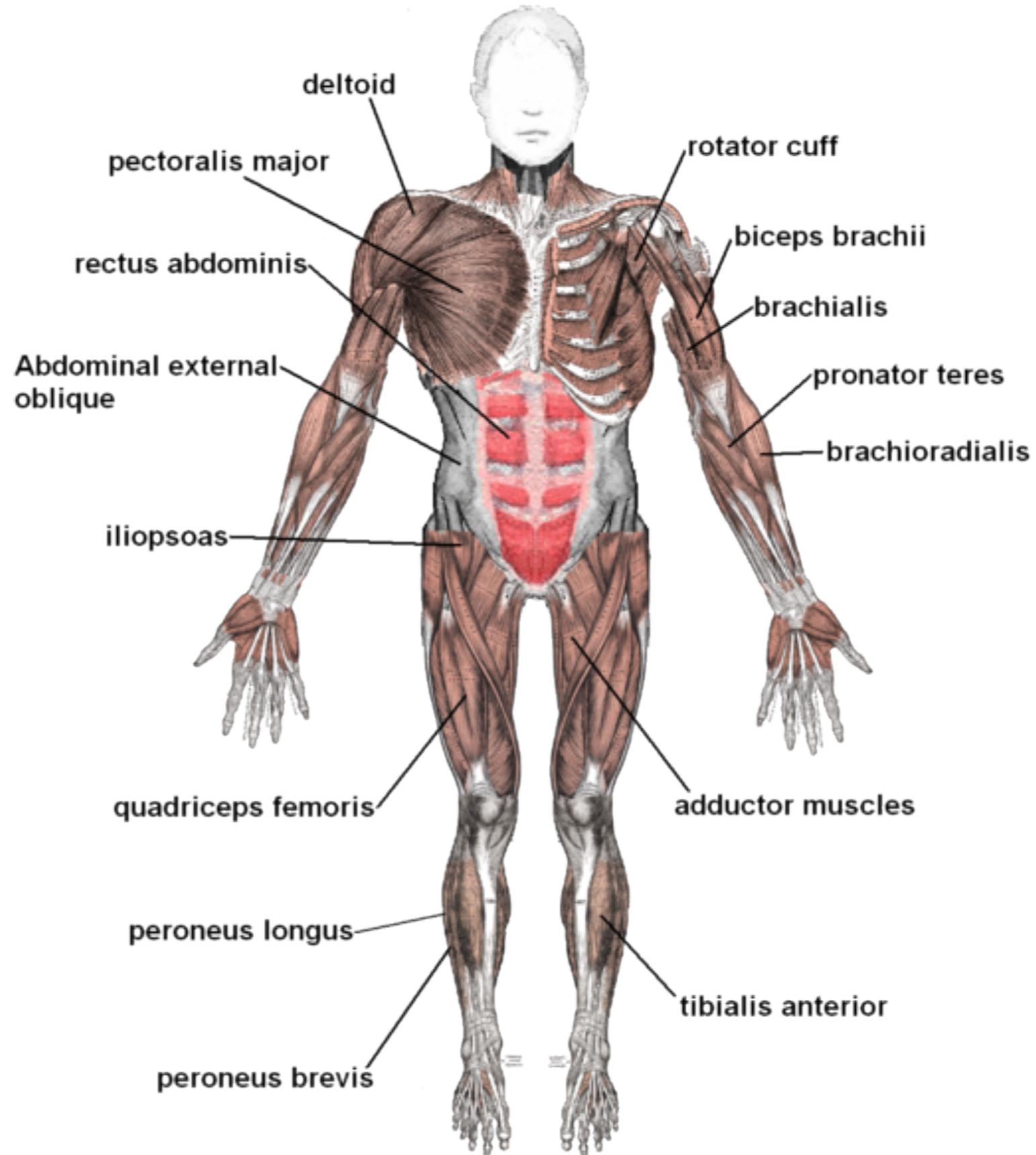
Likely she is focusing her attention on abstract task level objectives, and not the control of individual muscles, joints, etc.

# Human Body: many DOFS

206 bones

360 joints  
(230 movable  
joints)

600-800  
muscles



Source: Wikipedia

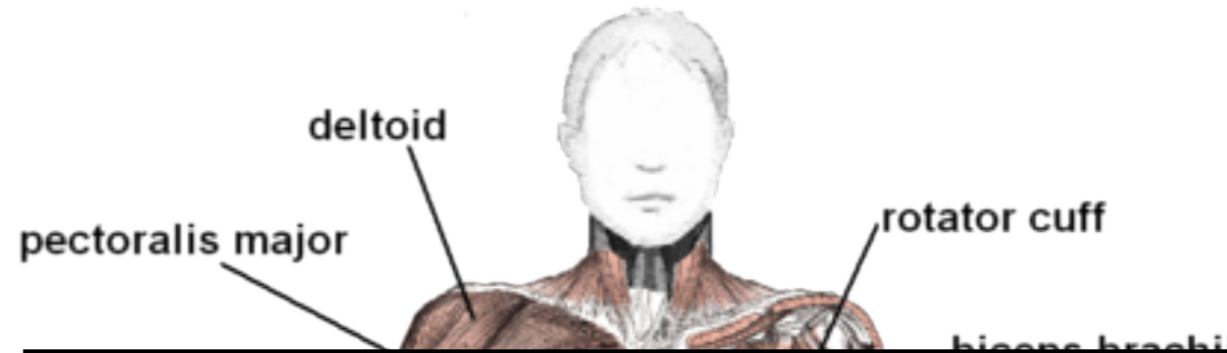
UNIVERSITY OF  
BIRMINGHAM

# Human Body: many DOFS

206 bones

360 joints  
(230 movable  
joints)

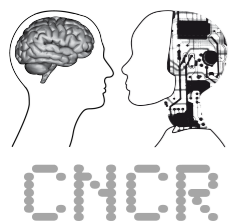
600-800  
muscles



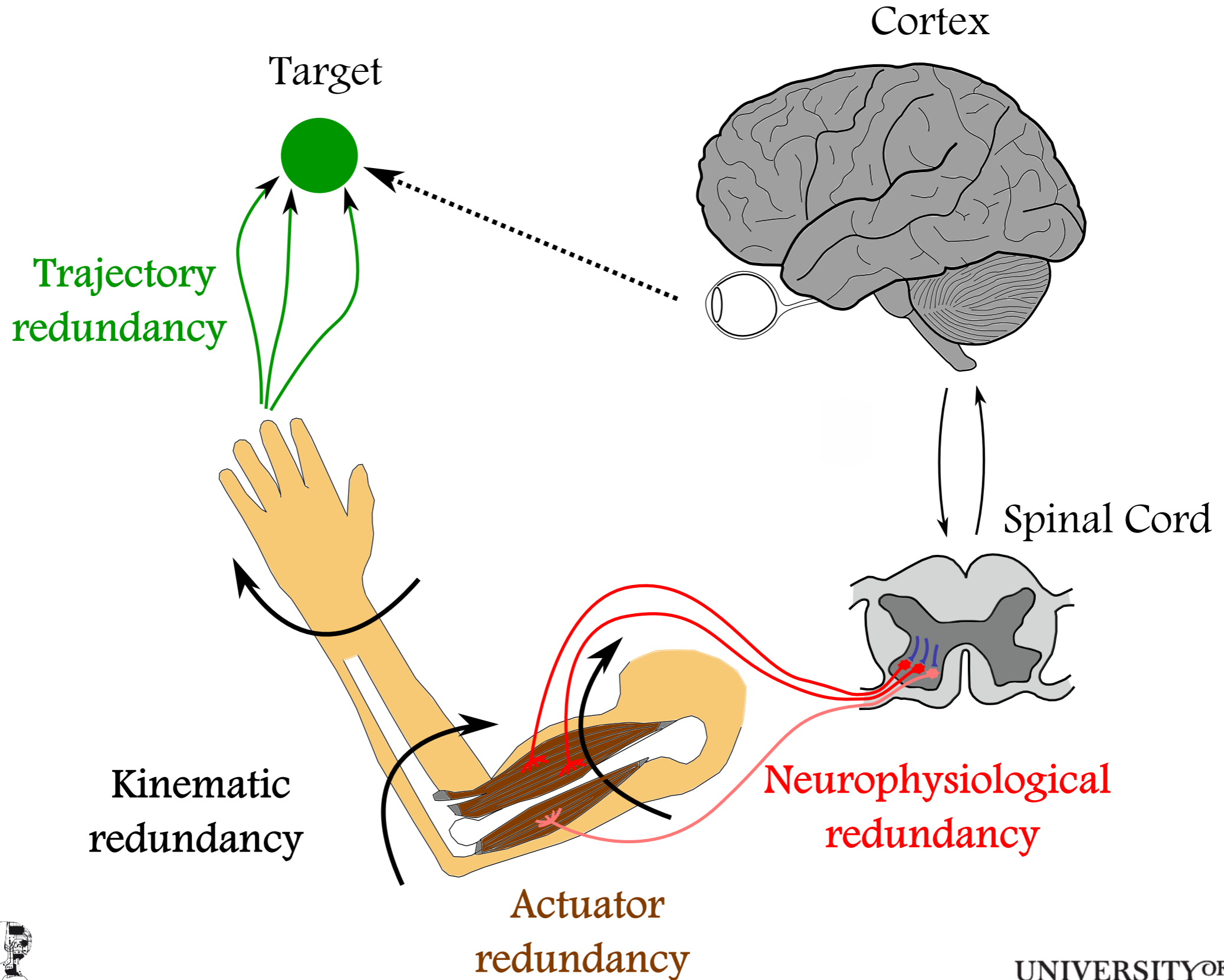
**Too much redundancy!**

Finding the optimal solution for 230 joints and 600 muscles in real-time is a computational nightmare.

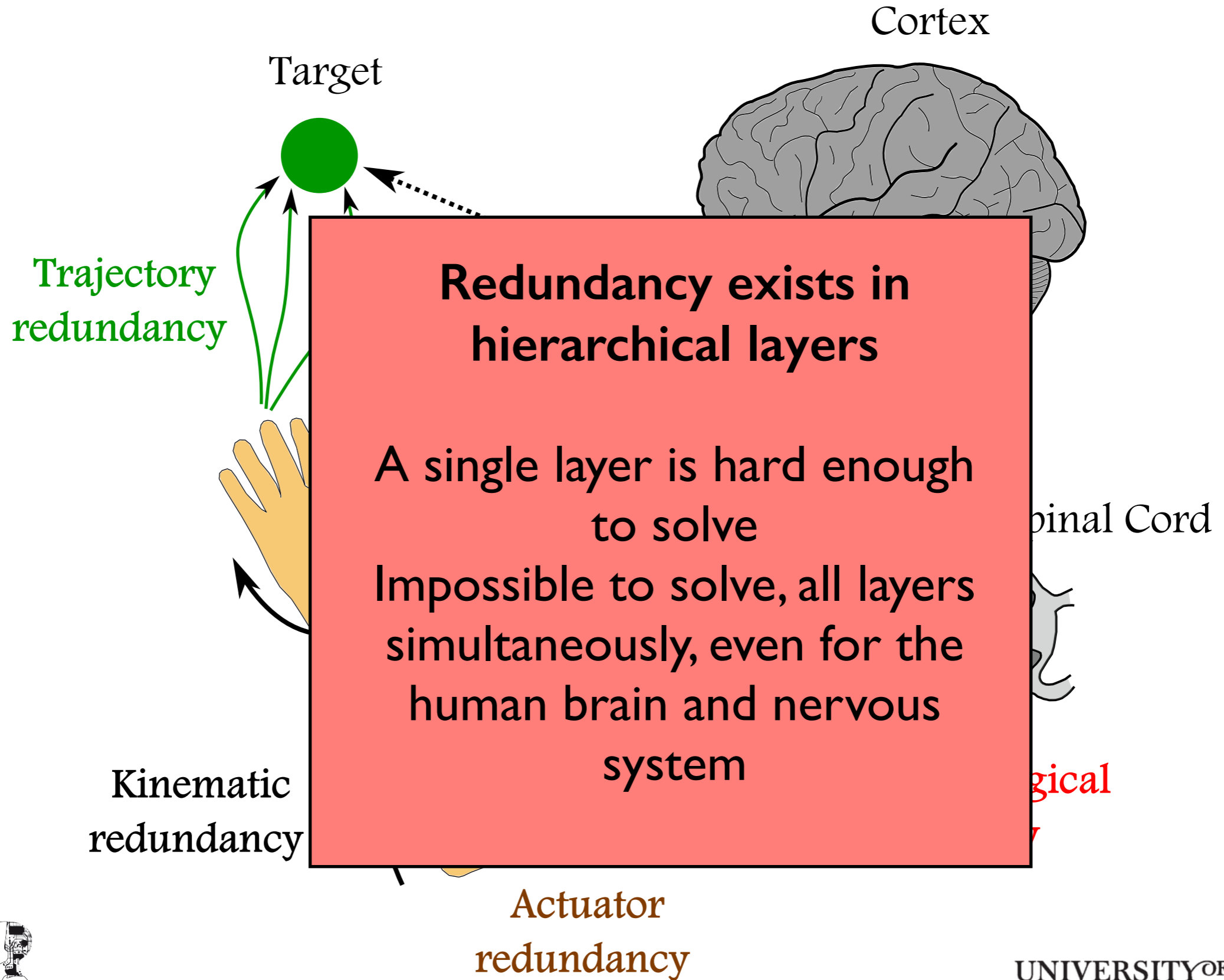
Even for a ~40 DOF torque controlled humanoid robot



# Types of redundancy in the motor system

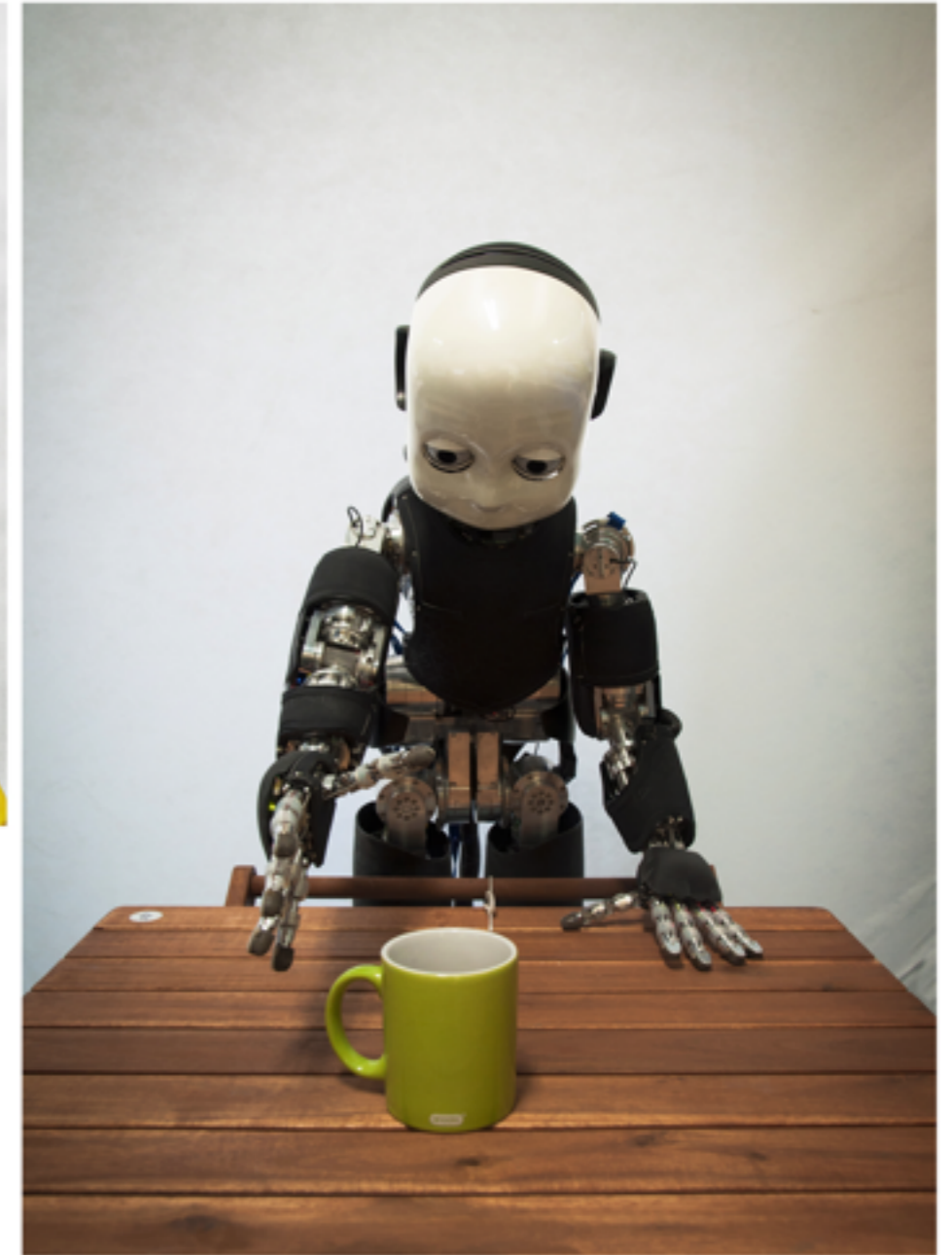


# Types of redundancy in the motor system

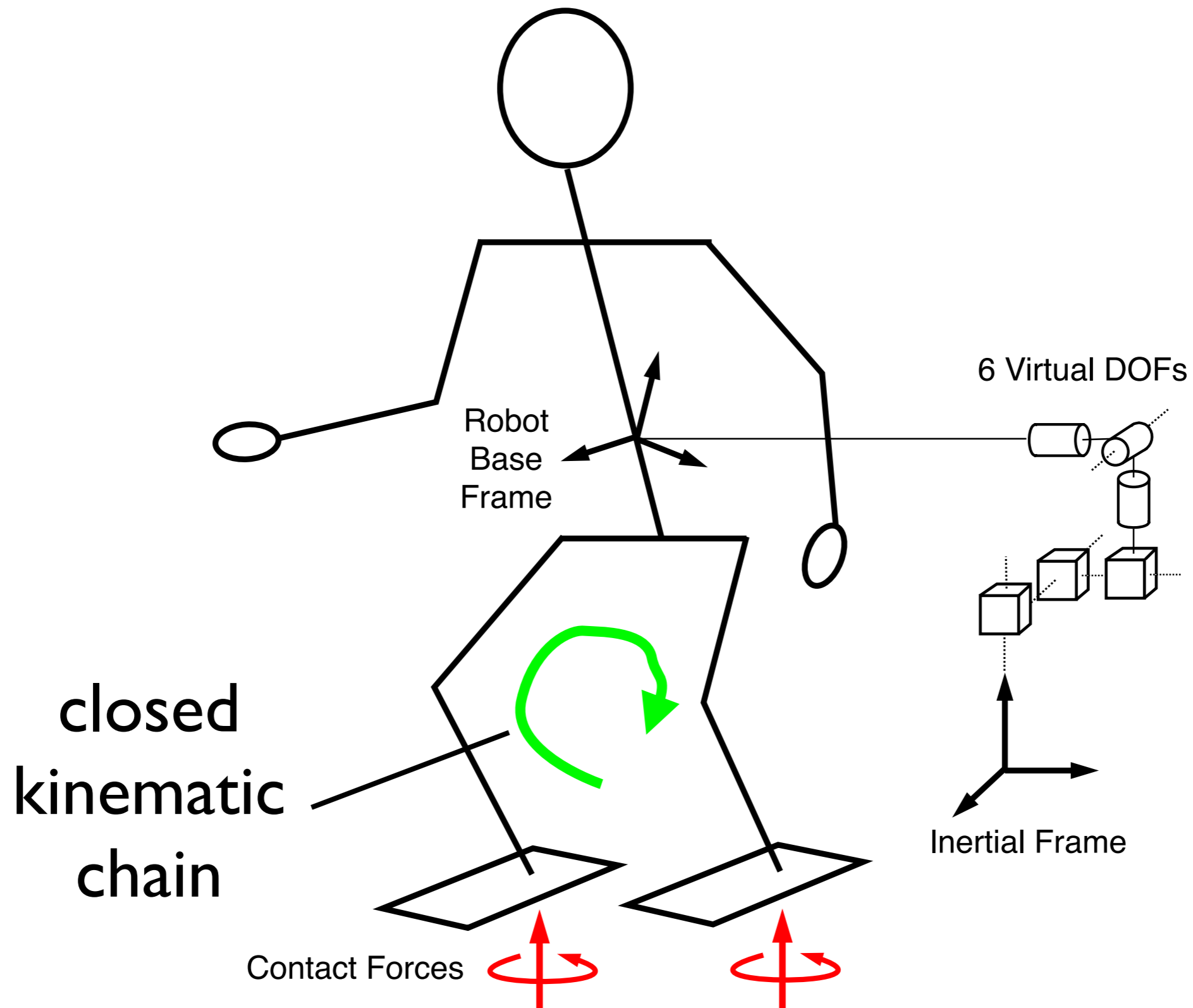




Even another type of redundancy:  
internal forces created by contact and closed  
kinematic chains



# Apparent in whole-body dynamics:





**Nikolai Bernstein  
(1896-1966)**



**Central Institute of  
Labour**

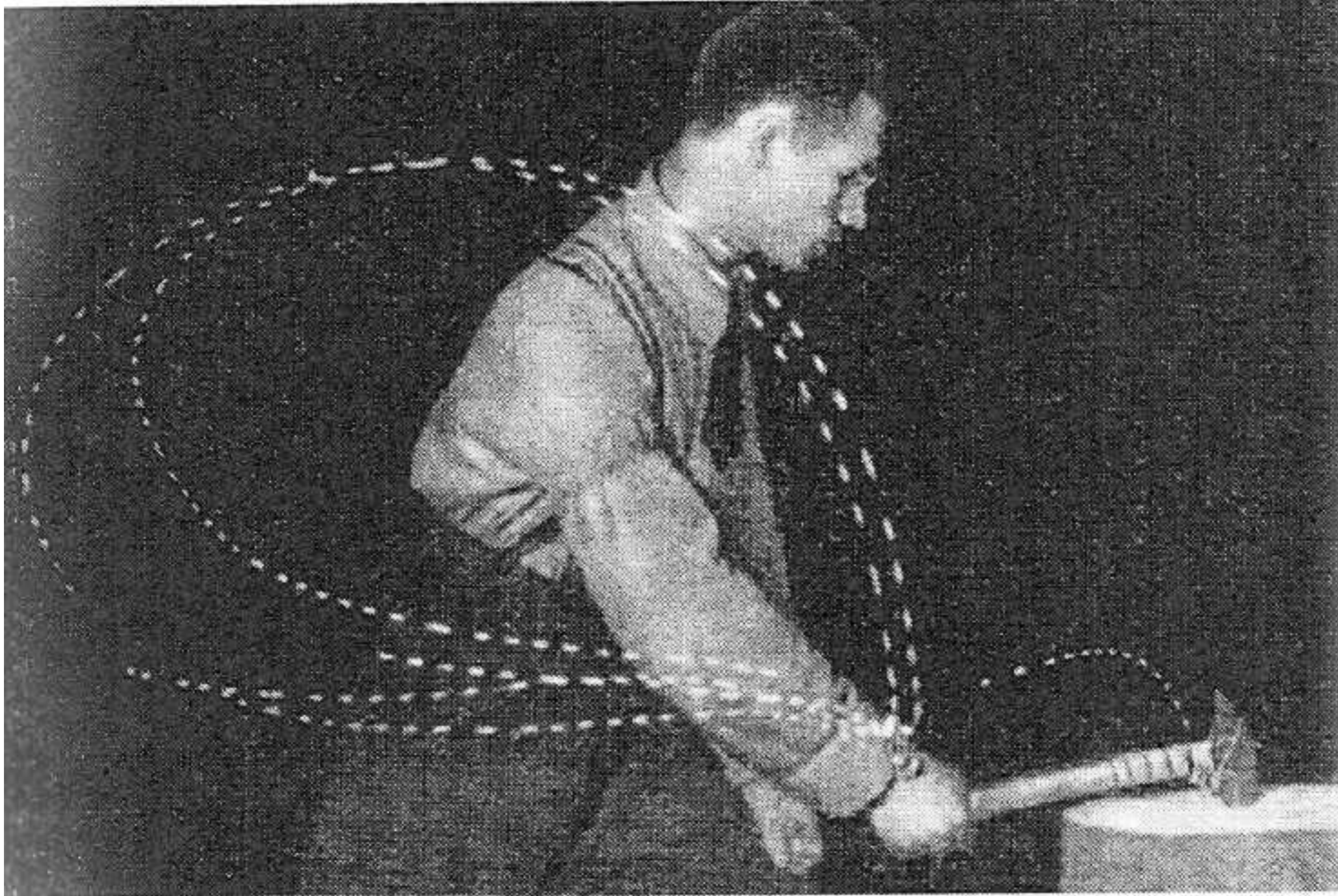
**One of the earliest researchers to study  
redundancy in the human motor system**

**НАУЧНЫЕ ОСНОВЫ  
ФИЗИЧЕСКОЙ  
КУЛЬТУРЫ**

**ОПЕРАТОР Н. ВИХИРЕВ**

**The Scientific Bases of Physical Culture  
Operator N. Vihirev**

# A Bernstein Cyclograph:





**Nikolai Bernstein  
(1896-1966)**

## Degrees of Freedom Problem:

"It is clear that the basic difficulties for co-ordination consist precisely in the extreme abundance of degrees of freedom, with which the [nervous] centre is not at first in a position to deal."

One solution to the Degree-of-Freedom Problem:  
Bernstein postulated that the nervous system may be functionally  
“freezing” joints to simplify the complexity.

Think about learning to ski:

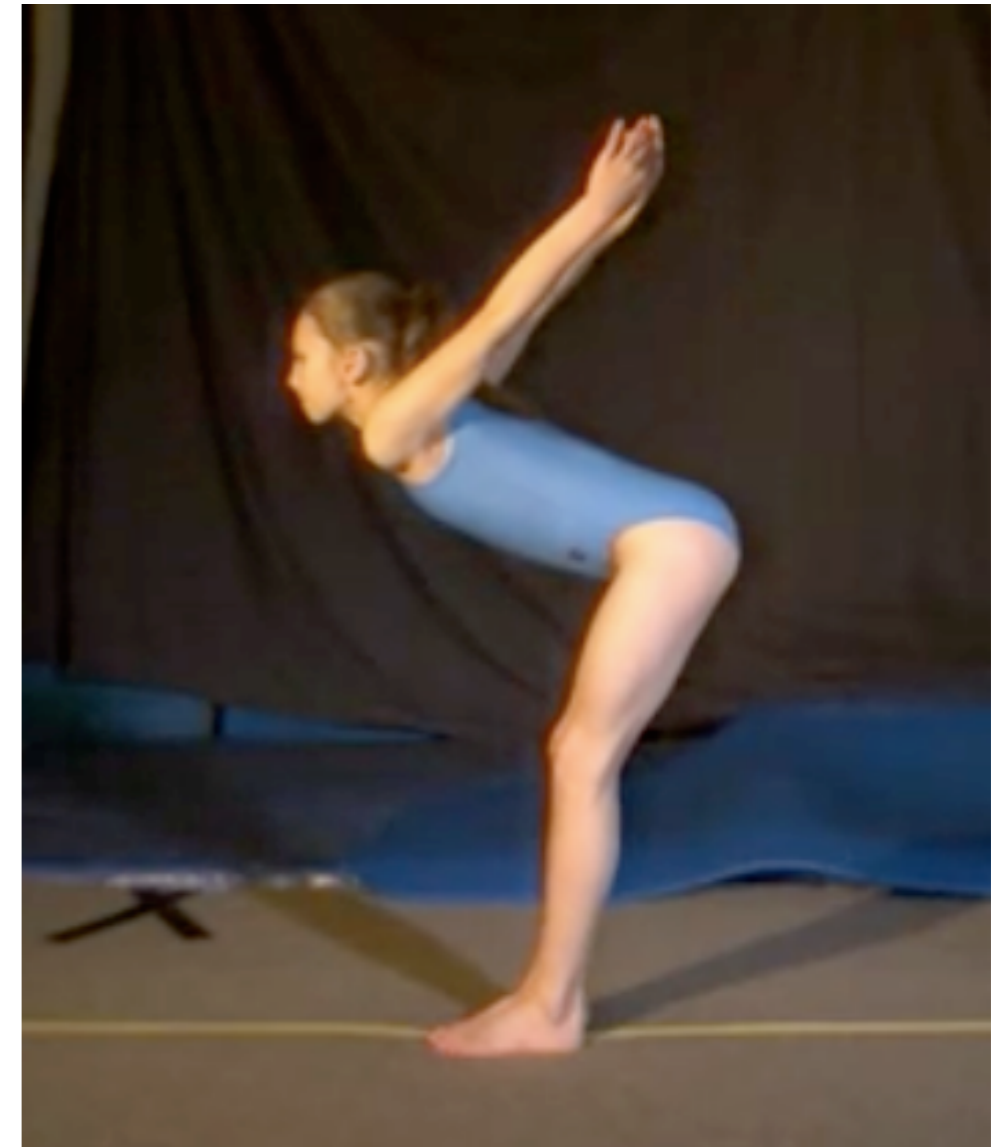


But really redundancy is quite useful:

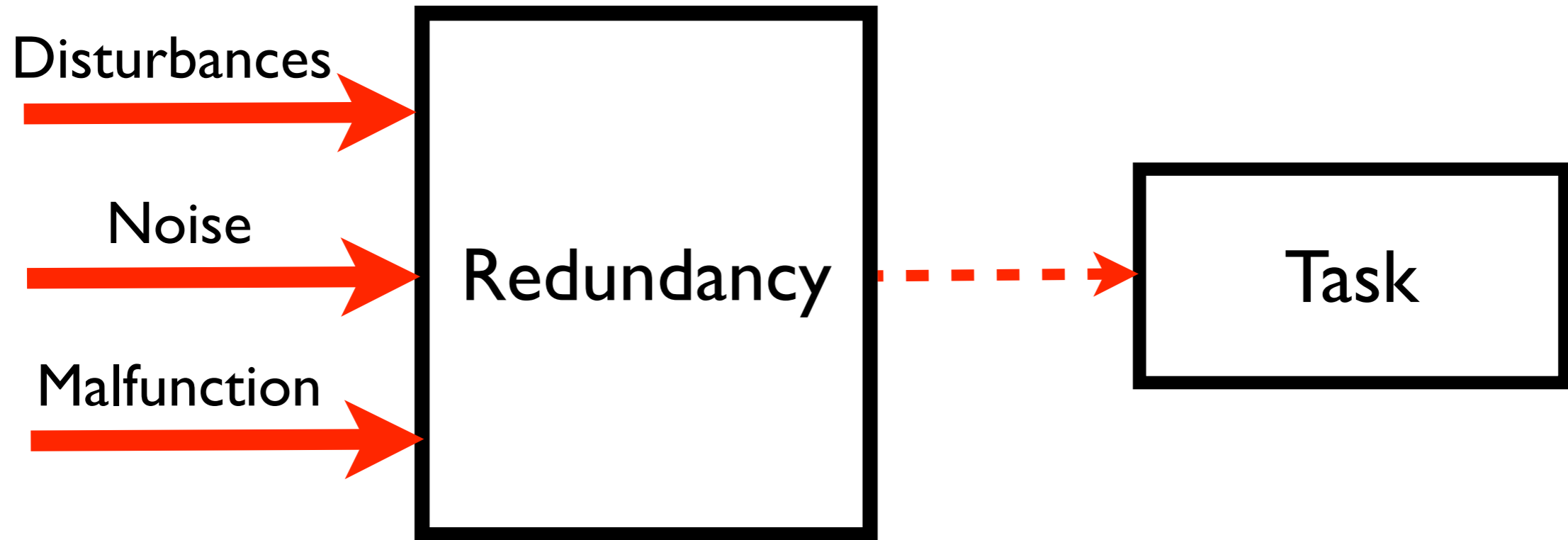


# Whole body motion, a challenging problem:

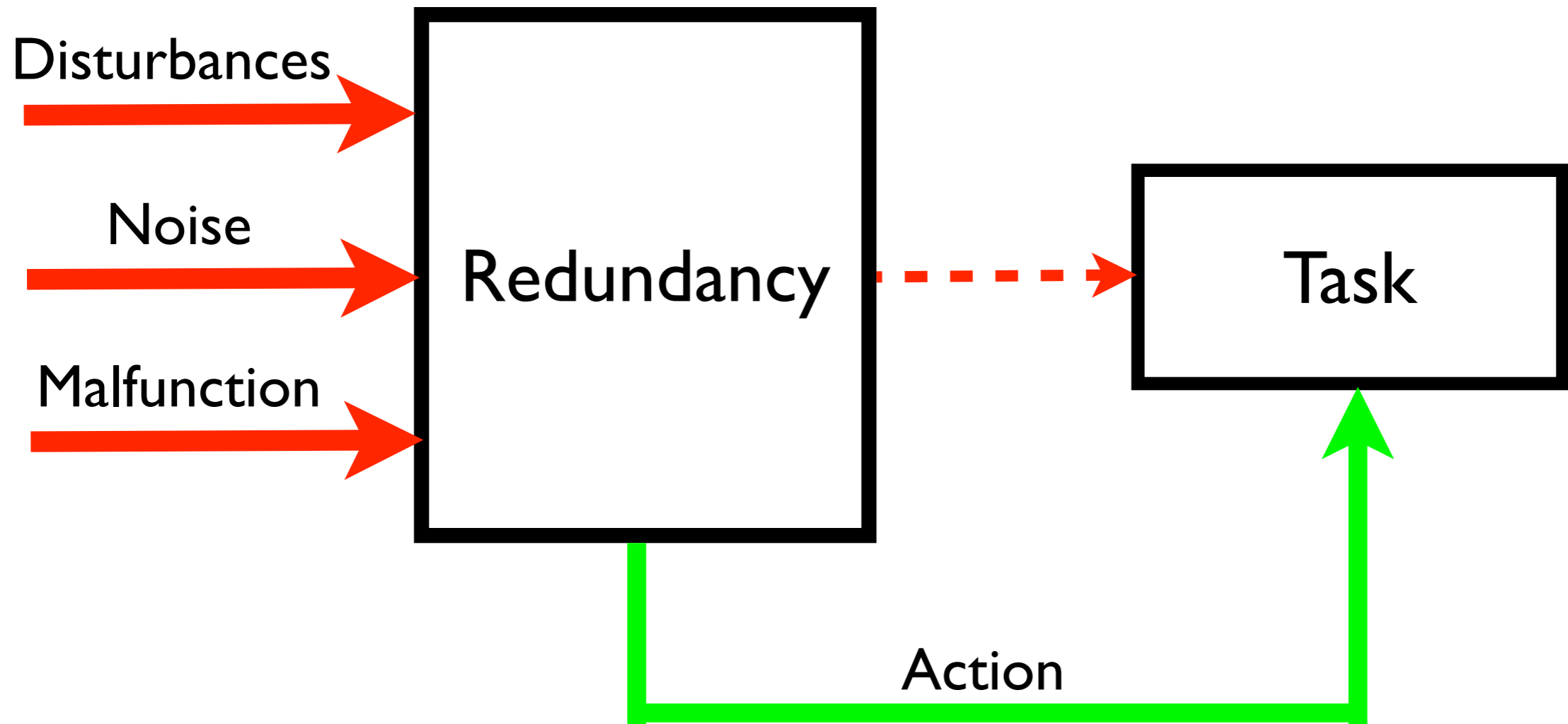
- High degree of freedom
- Highly dynamic
- High cost of failure
- Underactuated
- Internal Model?
- Planning required
- **Kinematically Redundant DOFs influence the task dynamics**



# Redundancy as “Filter” (or “Buffer”) protecting the task:



# Redundancy as “adding action”



# Operational Space Control (Khatib, 1987)

Assume rigid-body  
model of arm dynamics:

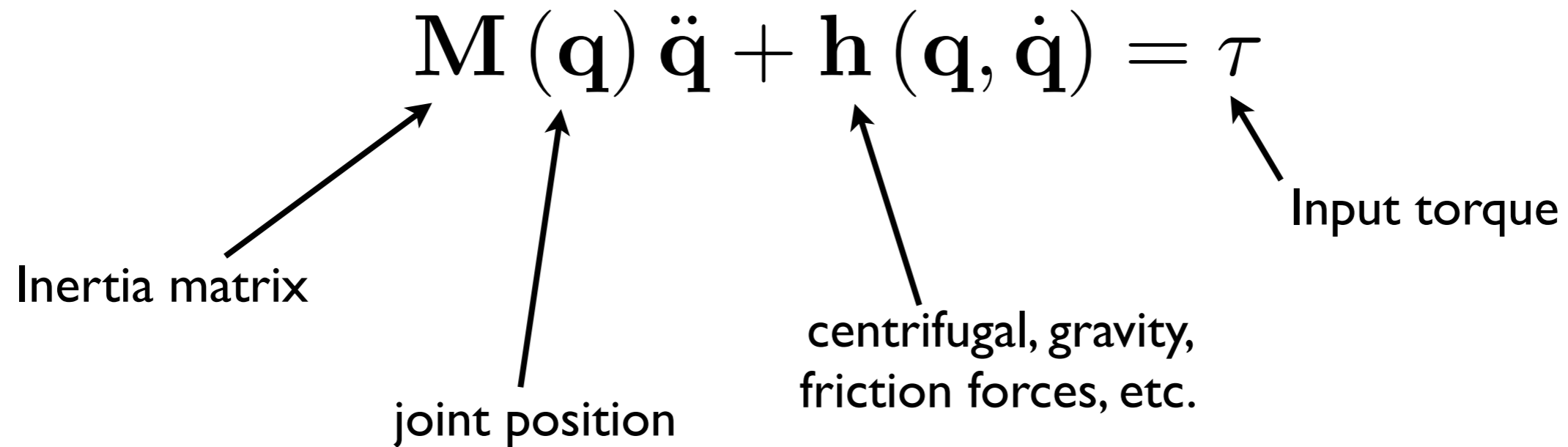
$$\mathbf{M}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}$$

Inertia matrix

joint position

centrifugal, gravity,  
friction forces, etc.

Input torque

The diagram shows the equation  $\mathbf{M}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}$  with four arrows pointing to its components. An arrow from 'Inertia matrix' points to  $\mathbf{M}(\mathbf{q})$ . An arrow from 'joint position' points to  $\mathbf{q}$ . An arrow from 'centrifugal, gravity, friction forces, etc.' points to  $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$ . An arrow from 'Input torque' points to  $\boldsymbol{\tau}$ .

# Operational Space Control (Khatib, 1987)

Rigid Body dynamics:  $\mathbf{M}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}$

Task:  $\mathbf{x} = f(\mathbf{q})$   
 $\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}$

We can *decouple* task and null-space forces:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{\text{task}} + \boldsymbol{\tau}_{\text{null}}$$

$$\boldsymbol{\tau} = \mathbf{J}^T \bar{\mathbf{J}}^T \boldsymbol{\tau} + (\mathbf{I} - \mathbf{J}^T \bar{\mathbf{J}}^T) \boldsymbol{\tau}$$

$$\bar{\mathbf{J}} = \mathbf{M}^{-1} \mathbf{J}^T (\mathbf{J} \mathbf{M}^{-1} \mathbf{J}^T)^{-1}$$

We have a tool to compute redundant  
*forces and dynamics*

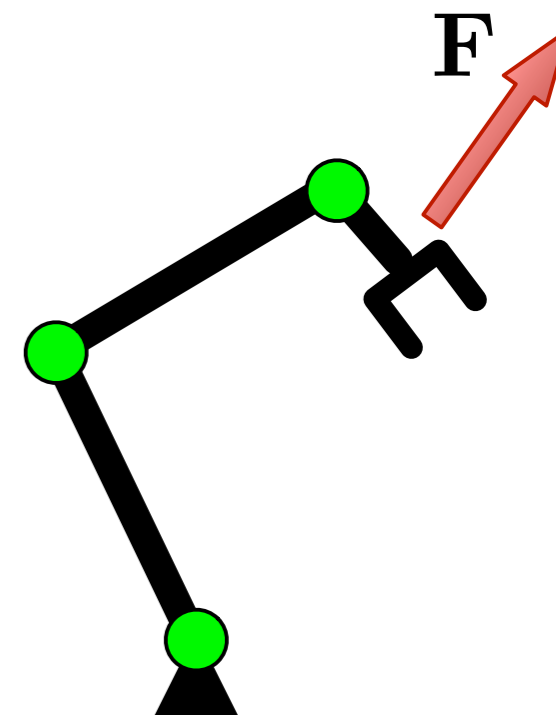
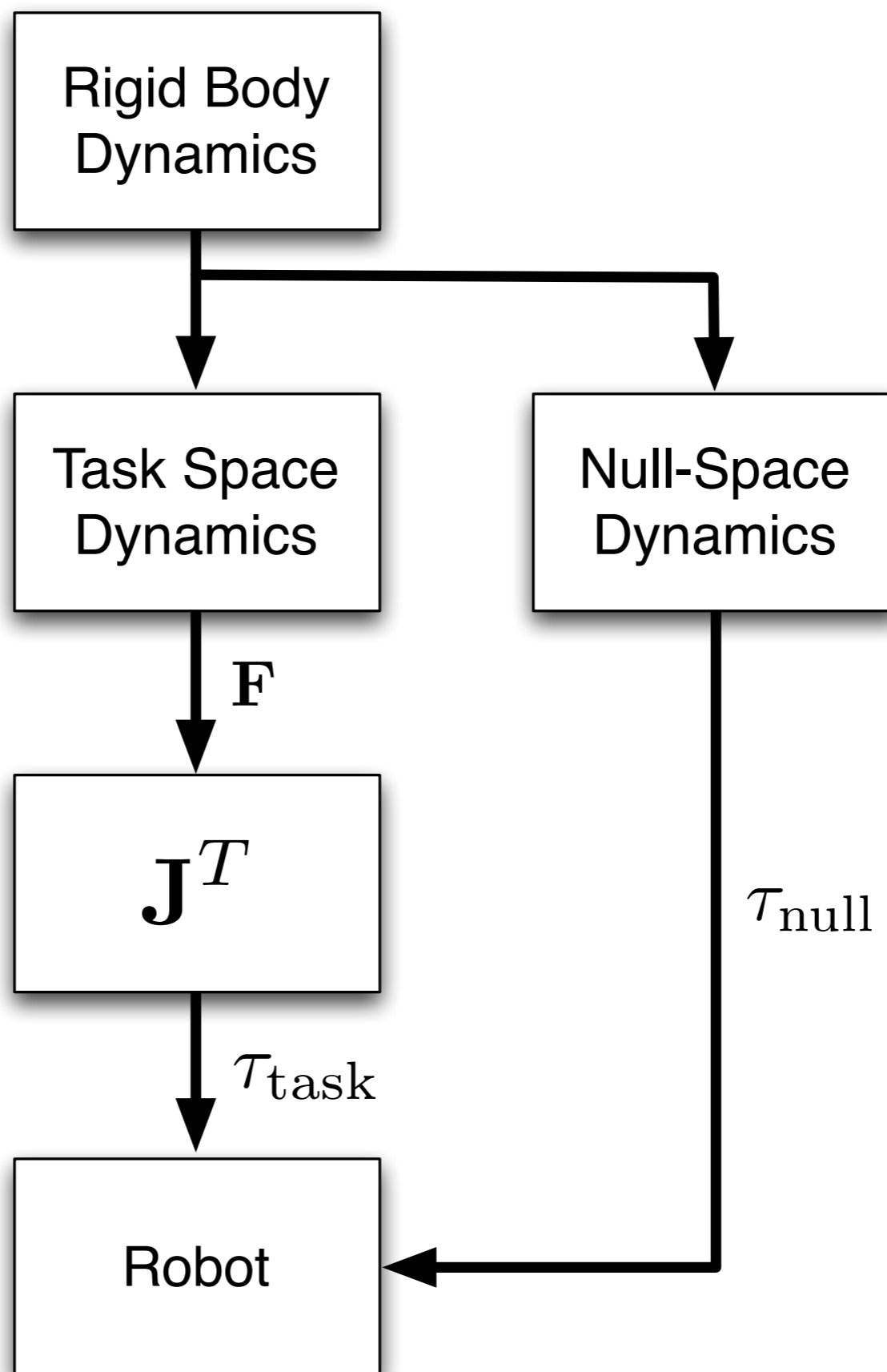
# Operational Space Control (Khatib 1987)

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}$$

$$\boldsymbol{\Lambda}\ddot{\mathbf{x}} + \boldsymbol{\Lambda}(\mathbf{J}\mathbf{M}^{-1}\mathbf{h} - \dot{\mathbf{J}}\dot{\mathbf{q}}) = \mathbf{F}$$

$$\boldsymbol{\Lambda} = (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)^{-1}$$

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{F}$$



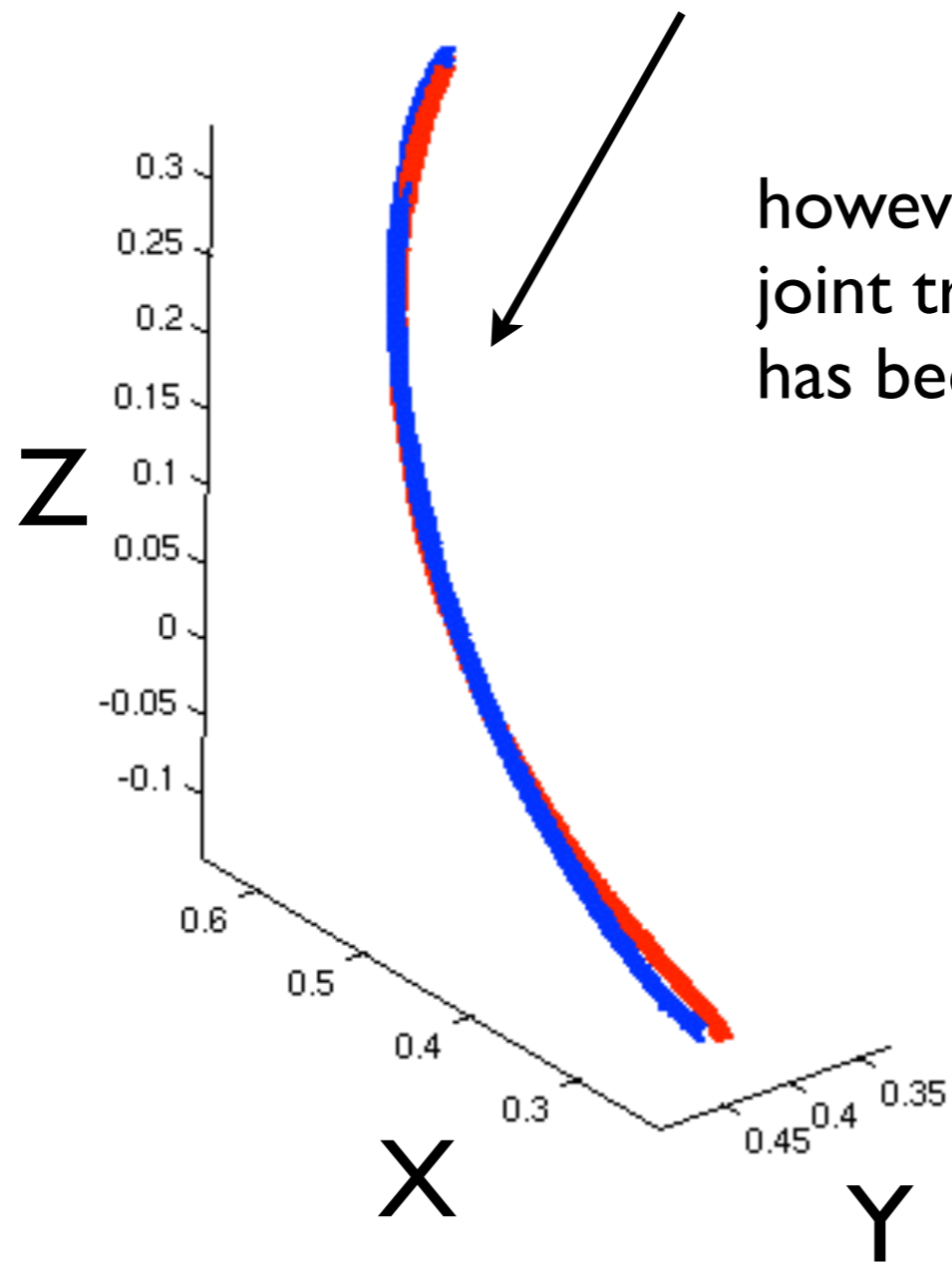
# Could the Brain be doing some form of Operational Space Control?



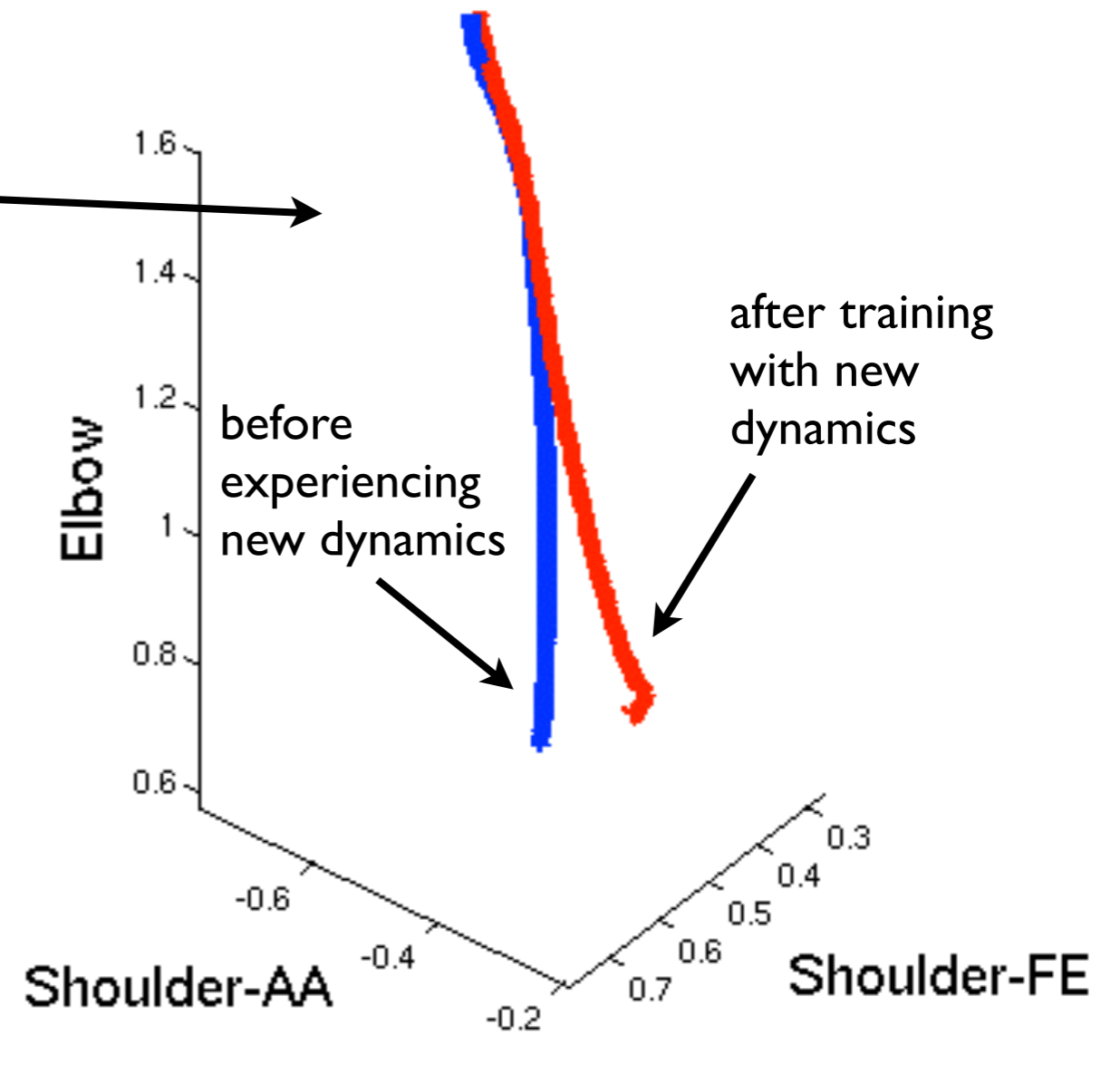
Mistry, M.;Mohajerian, P.;Schaal, S. (2005). Arm movement experiments with joint space force fields using an exoskeleton robot, *IEEE Ninth International Conference on Rehabilitation Robotics*, pp.408-413

# Result:

subjects learn to compensate for the extraneous dynamics, and (after sufficient training) return hand paths to nominal

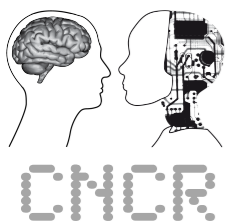


however, their joint trajectory has been altered



Hand Trajectory

Joint Trajectory





How to explain what's happening?

The hypothesis: subjects are only learning internal models of *task-relevant* components of the force field

# Operational Space Control

Formulate the arm controller as an operational space controller:

$$\tau = \mathbf{J}^T \bar{\mathbf{J}}^T \left( \hat{\mathbf{M}} \mathbf{J}^+ \left( \ddot{\mathbf{x}}_d - \dot{\mathbf{J}} \dot{\mathbf{q}} \right) + \mathbf{D} \dot{\mathbf{q}} \right) + \hat{\mathbf{h}} + K_P (\mathbf{q}_d - \mathbf{q}) + K_D (\dot{\mathbf{q}}_d - \dot{\mathbf{q}})$$

desired task trajectory

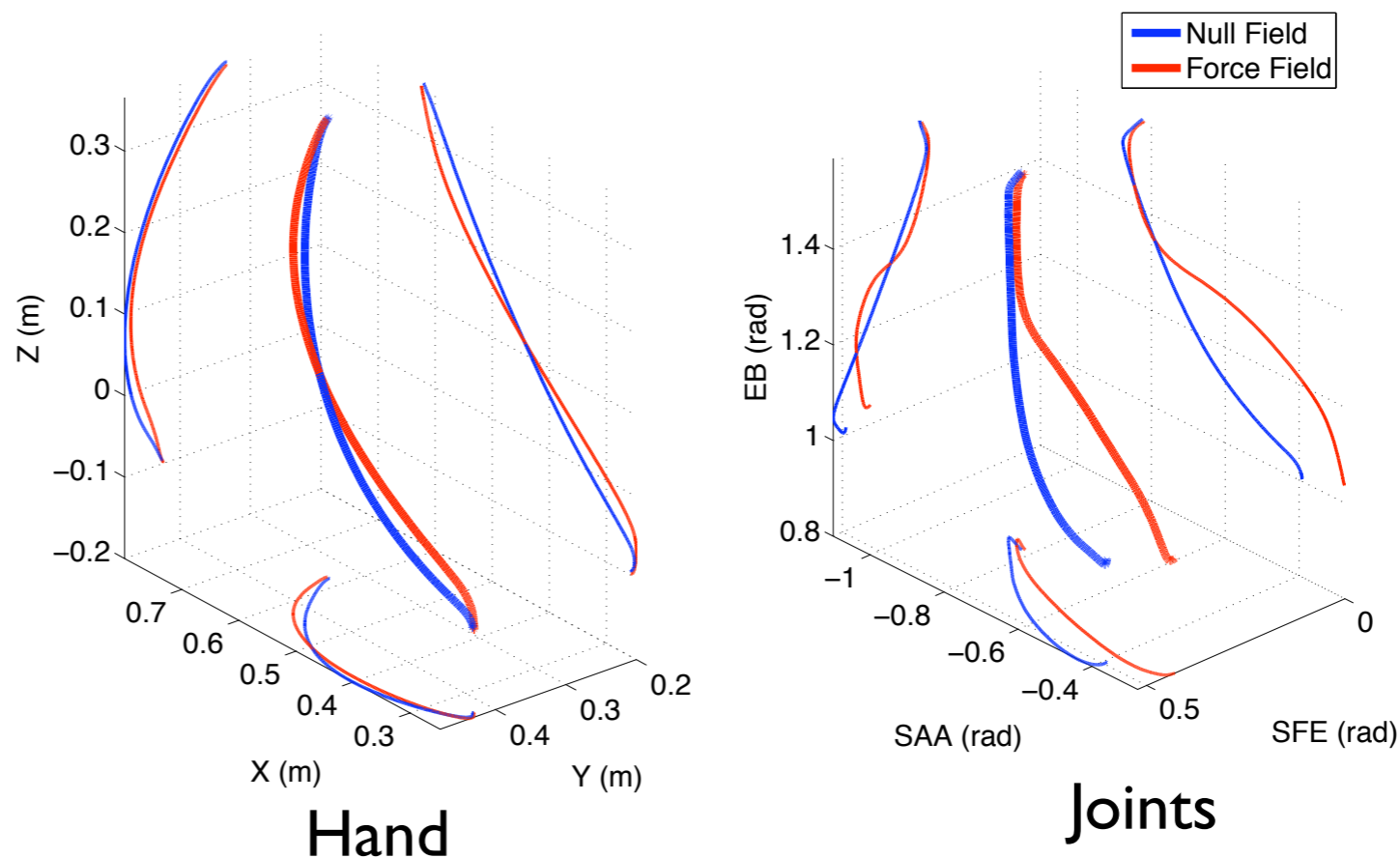


Only the task relevant forces of the force field are compensated.



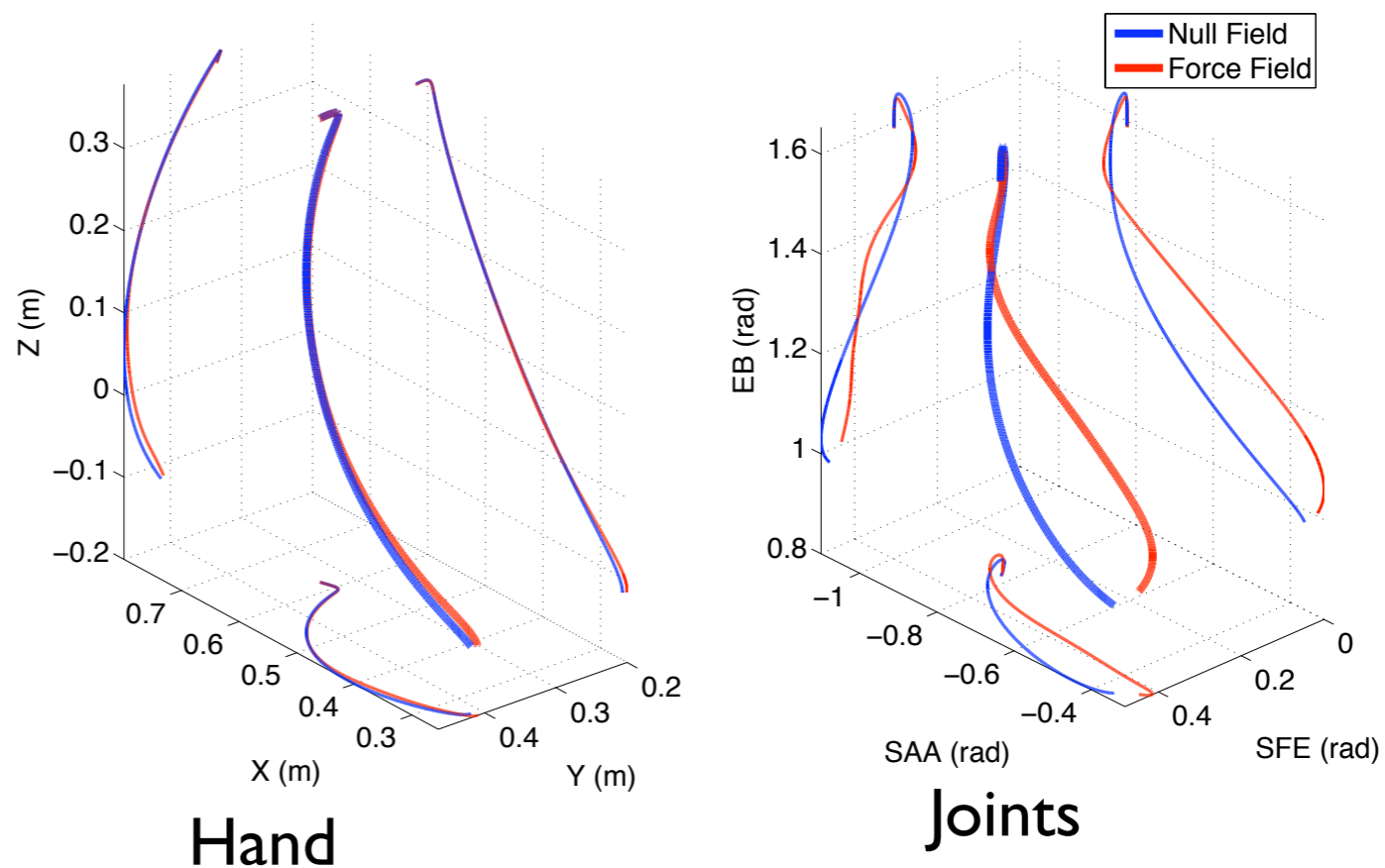
Humans only learn and compensate for the *task relevant* component of the novel dynamics.

Actual data from subject:



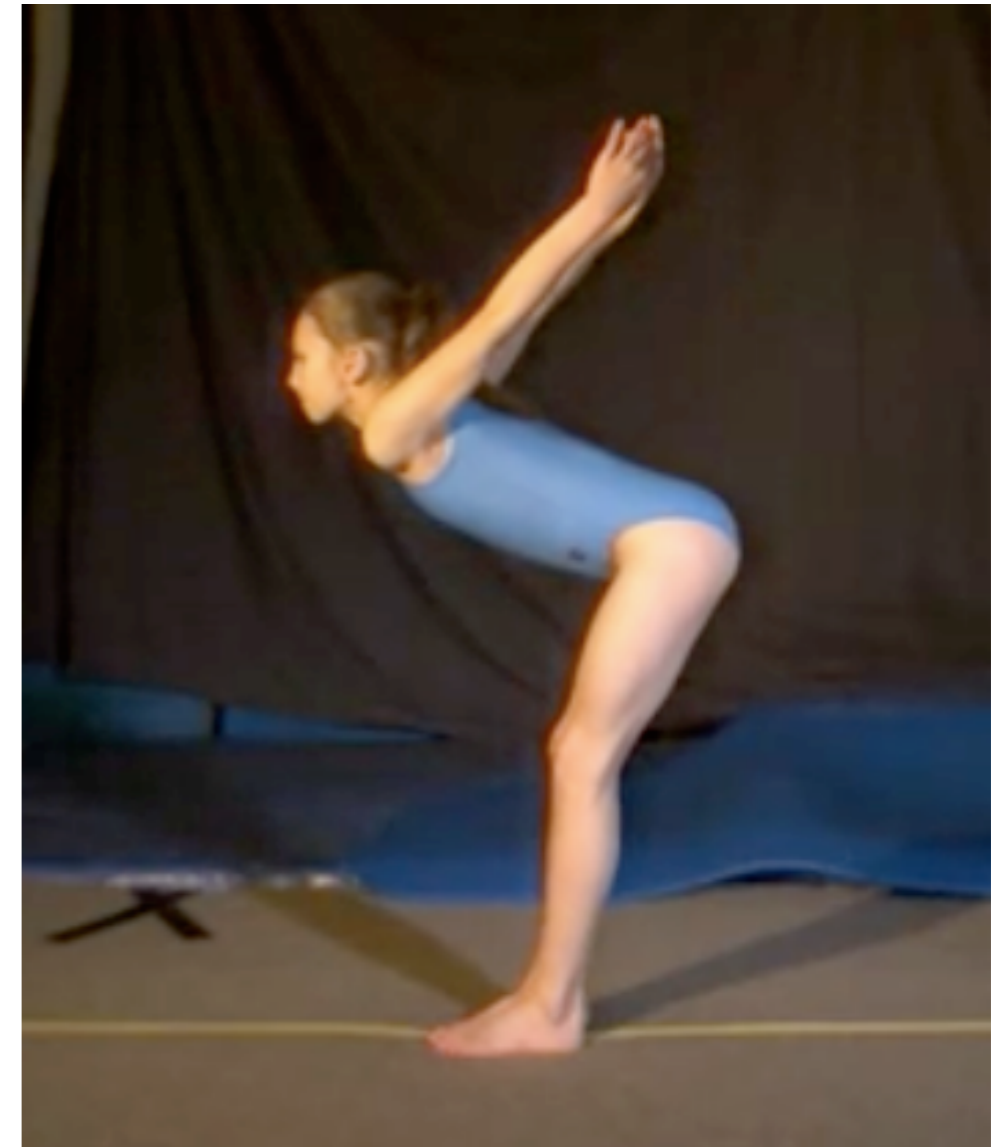
We test this hypothesis in simulation, using a model of arm dynamics and a *task-space* controller

Simulated results:

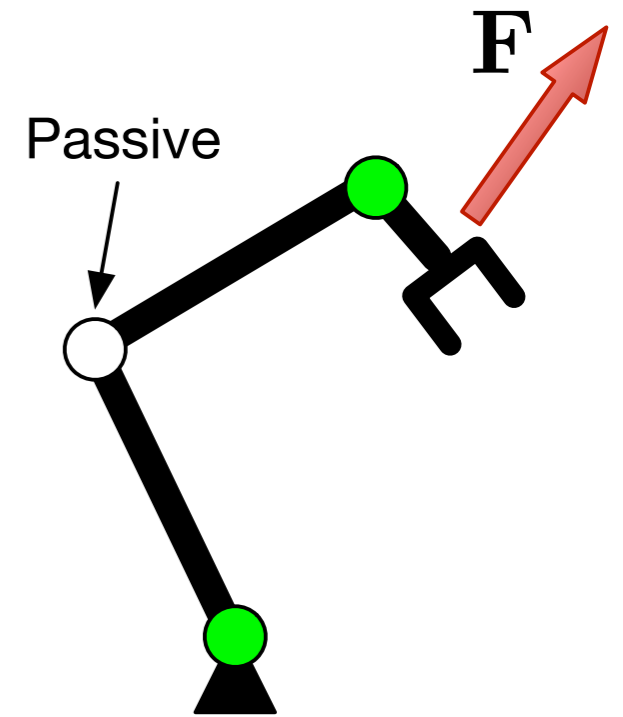
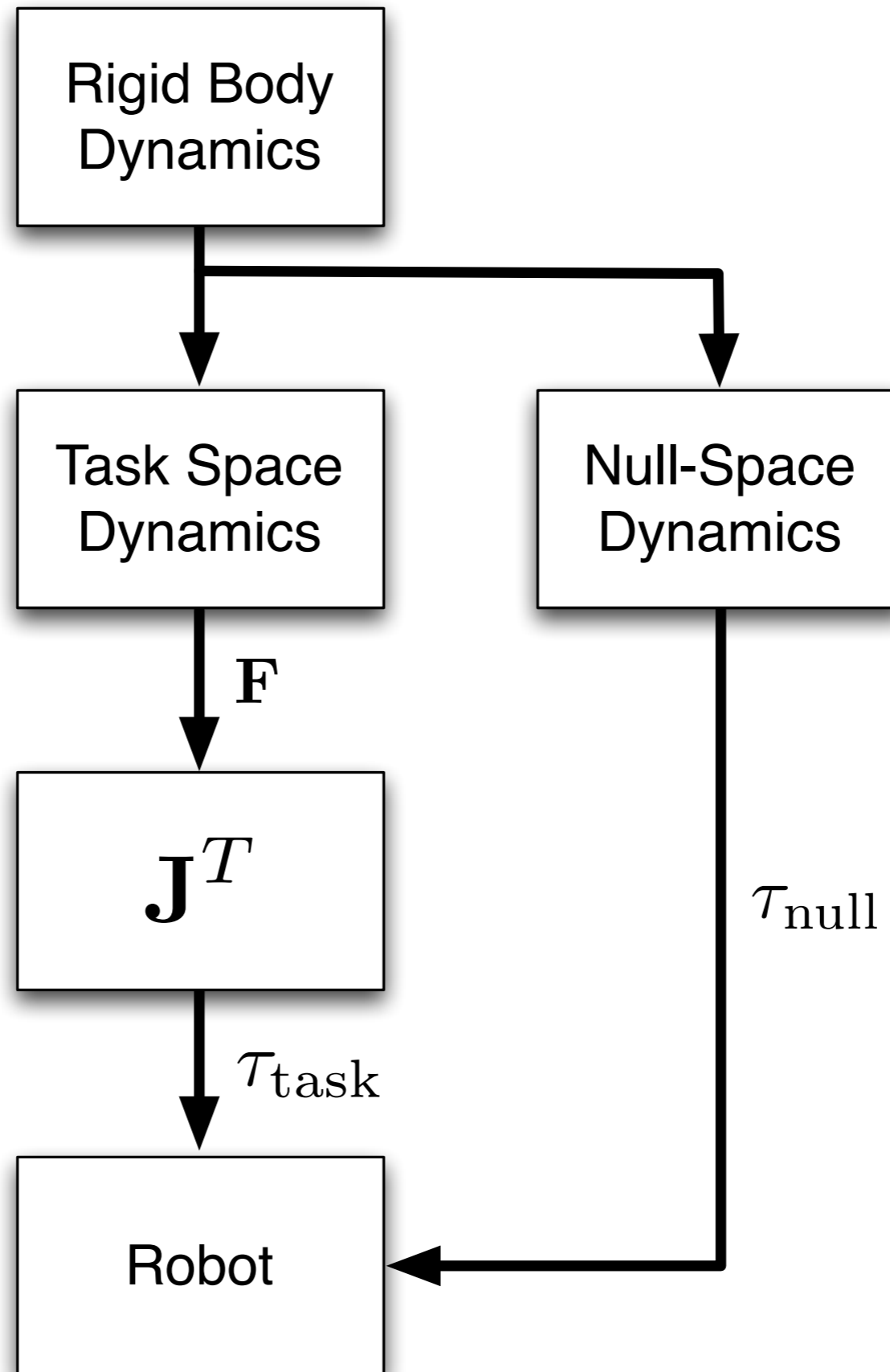


# Back to Gymnastics:

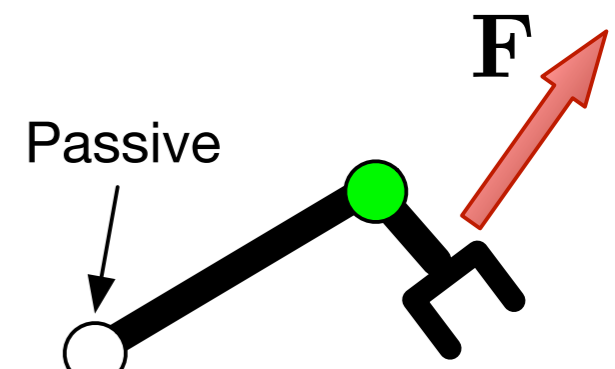
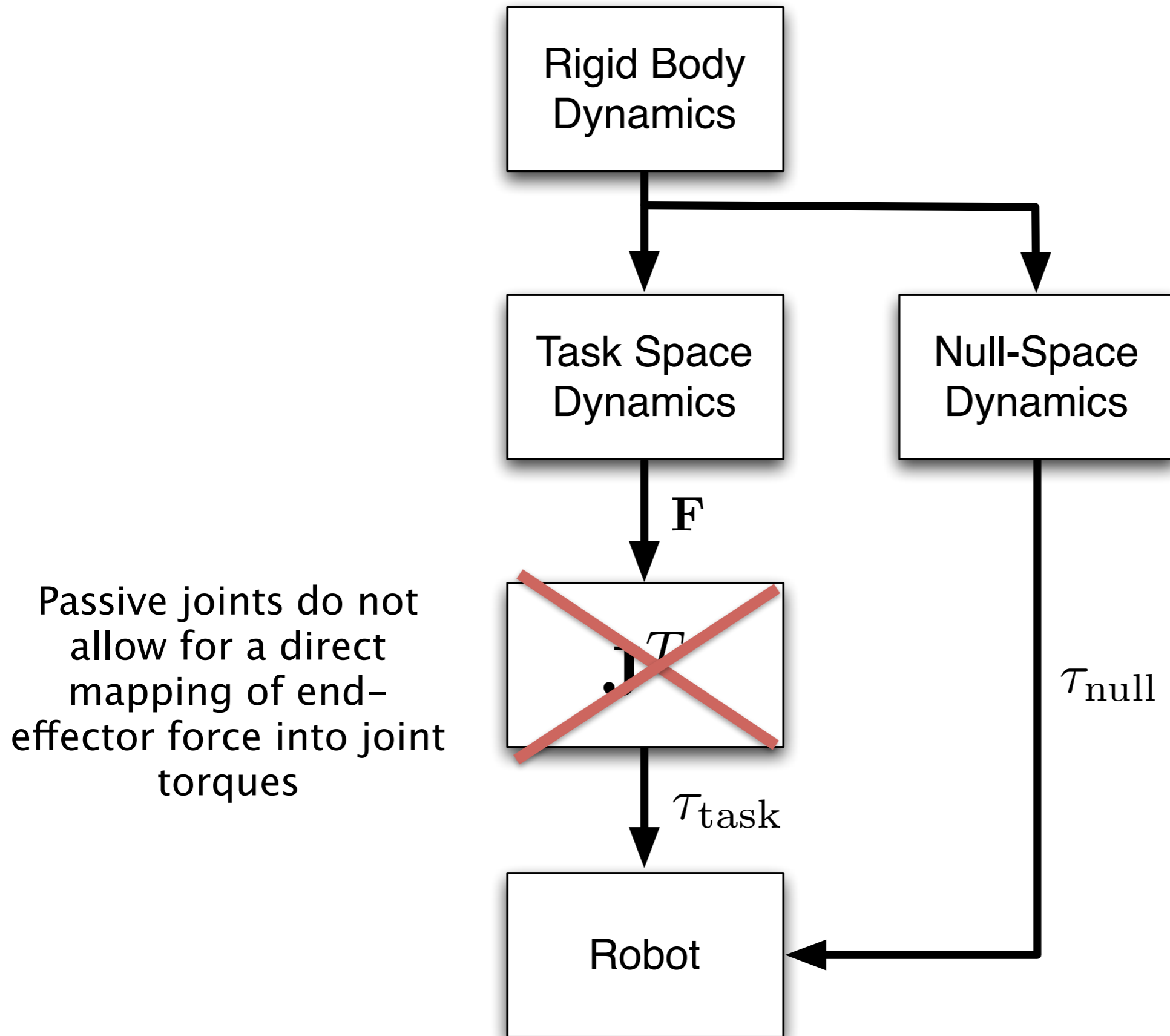
- High degree of freedom
- Highly dynamic
- High cost of failure
- **Underactuated**
- Internal Model?
- Planning required
- **Kinematically Redundant DOFs influence the task dynamics**



# Underactuated Operational Space Control

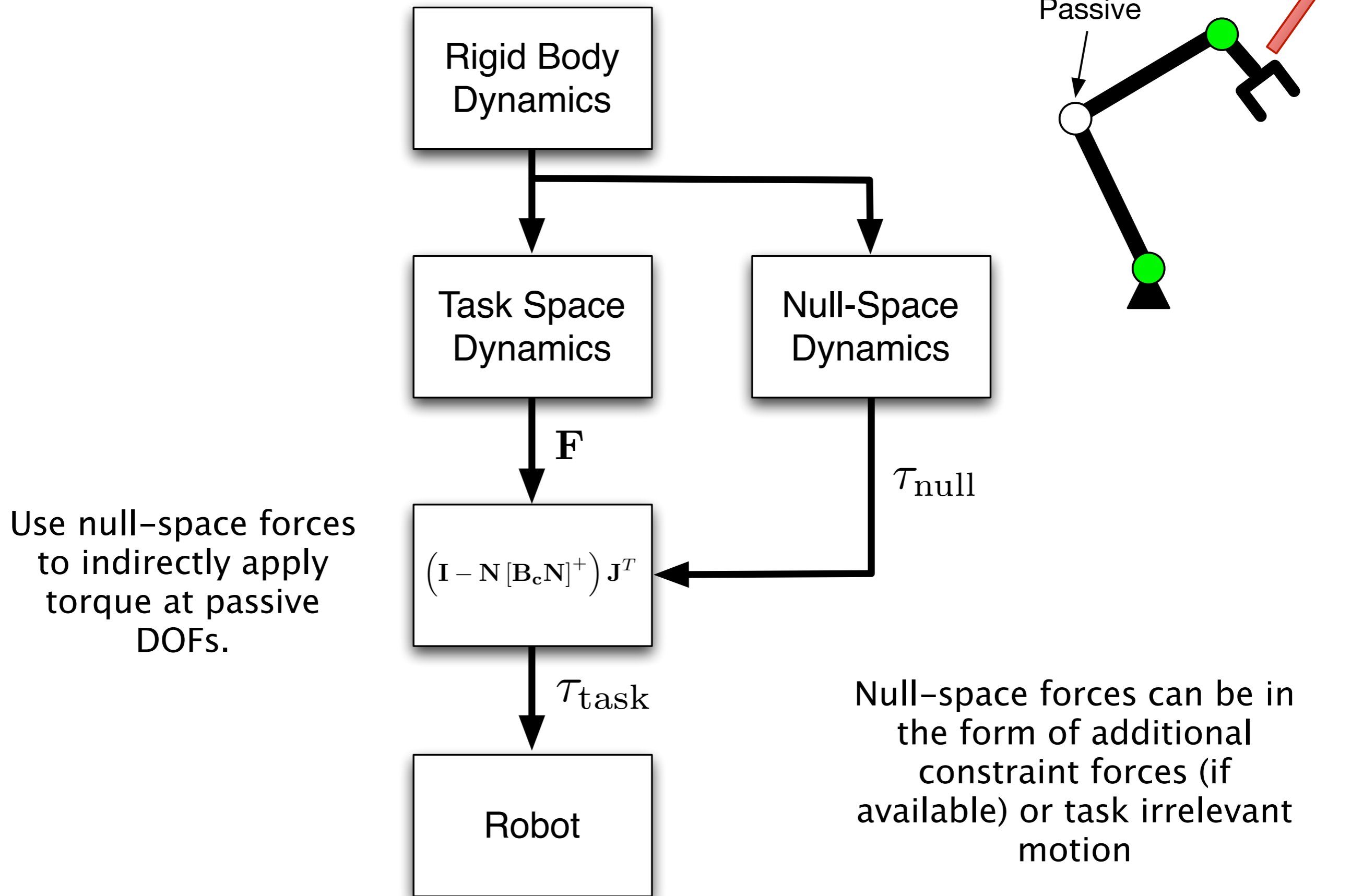


# Underactuated Operational Space Control



Passive joints do not allow for a direct mapping of end-effector force into joint torques

# Underactuated Operational Space Control



# Underactuated Operational Space Control

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{F} + \mathbf{N} \tau_0 \quad \boldsymbol{\tau} = \mathbf{B} \boldsymbol{\tau}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{J}^T \mathbf{F} + \mathbf{N} \tau_0 = \mathbf{B} \mathbf{J}^T \mathbf{F} + \mathbf{B} \mathbf{N} \tau_0$$

$$\tau_0 = - [(\mathbf{I} - \mathbf{B}) \mathbf{N}]^+ (\mathbf{I} - \mathbf{B}) \mathbf{J}^T \mathbf{F}$$

$$\boldsymbol{\tau} = \left( \mathbf{I} - \mathbf{N} [(\mathbf{I} - \mathbf{B}) \mathbf{N}]^+ \right) \mathbf{J}^T \mathbf{F}$$

Generate torque at passive DOFs via inherent dynamic coupling  
Similar to Partial Feedback Linearization



# Planning behaviors

Operational Space Control structures the problem, such that we can search for a solution in the lower dimensional task space:

$$\tau = \left( \mathbf{I} - \mathbf{N} [(\mathbf{I} - \mathbf{B}) \mathbf{N}]^+ \right) \mathbf{J}^T \mathbf{F}$$

search in  $\mathbf{F}$  “space” instead of  $\tau$



# One interesting application: control of an “floating base” manipulator for nuclear decommissioning

**BROKK**<sup>®</sup>



National Nuclear Laboratory



## Conclusions:

Too many DOFs in the human body

Solving redundancy is a computational “problem”

However this redundancy can also be exploited, *to assist the task.*

Thinking about the structure of these problems, e.g. the operational space, helps us do this.