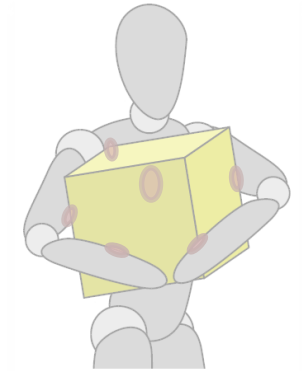


Optimal Contact Force Distribution In Whole-Body Loco-Manipulation Tasks



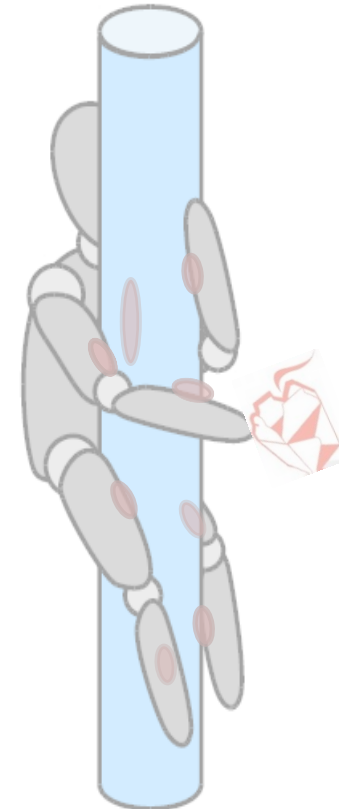
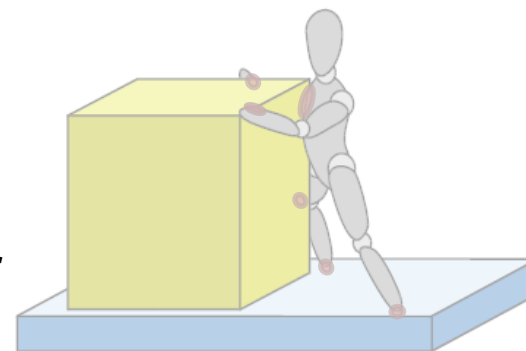
Edoardo Farnioli, Marco Gabiccini, Antonio Bicchi



University of Pisa



Italian Institute of
Technology



Outline

Review some basic notions of grasping

- Form/Force Closure
- Grasping with and without Hands
- Whole-Hand & Underactuated Grasping

Loco-Manipulation

- Floating frames
- A unified formulation of Whole-Body Loco-Manipulation

Optimization of Contact Force Distribution

- A Local Convexity Result
- A Fast-Converging Loco-Manipulation Optimization algorithm
- Simulations and Experiments

Grasping with and without a hand

Are hands important for grasping?

In what specific sense?

What notions are hand-independent, and what are hand-specific?

Locomotion with and without a body

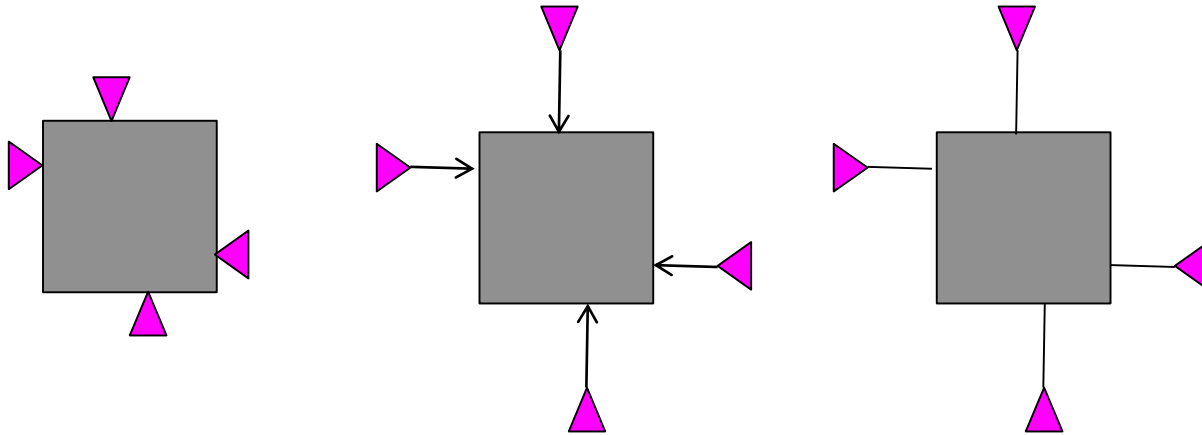
Are bodies important for locomotion?

In what specific sense?

What notions are body-independent, and what are body-specific?

Immobilization: Form Closure

Form-closure: the ability to prevent motions of the constrained object, relying only on unilateral (contact) constraints.



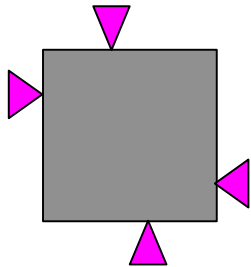
Pins at the contact points indicate that only motions of the object that cause penetration of the pin in the object are prevented by that constraint.

Reuleaux [1875]: 2D FmC \rightarrow at least 4 contact points

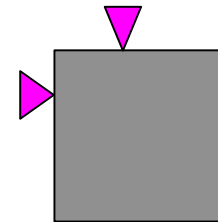
Somov [1900]: 3D FmC \rightarrow at least seven contact points

Form Closure: Definition

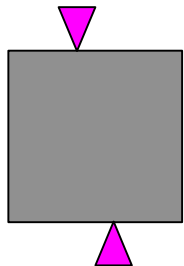
Definition: *A set of contact constraints is FORM CLOSURE if there exist no object motion (twist) which does not violate at least one constraint*



Form Closure



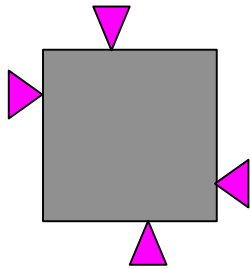
Not Form Closure



Partial Form Closure

Form Closure: Definition

Definition: A set of contact constraints is FORM CLOSURE if every object motion (twist) violates at least one constraint



A Linear Programming Problem:

$$\begin{cases} \text{Maximize } \mathbf{f}^T \mathbf{x} \\ \text{subject to } \mathbf{N}^T \mathbf{G}^T \mathbf{x} \geq 0 \end{cases}$$

The existence of a feasible solution for this problem is a necessary and sufficient condition for negating the form-closure property.

The LP formulation lends itself to algorithmic techniques, such as dualization [Cheng & Orin 1990]

NB: Feasibility only is important. Vector $\mathbf{f} \in \mathbb{R}^d$ is arbitrary and inessential. If $\mathbf{f} \in \mathbb{R}^d$ is interpreted as a force, \mathbf{x} could be regarded as the corresponding object velocity [Trinkle, 1992]).

Strictly speaking, the concept of "force" is inessential to form-closure and can be altogether avoided in its treatment.

Force Closure

The notion of force-closure is less unanimous.

The intuitive meaning of FcC is that *motions of the grasped object are (completely or partially) restrained despite whatever external disturbance, by virtue of suitably large contact forces that the constraining device (the end-effector) is capable to exert on the object.*

Hence, **FcC** differs from form-closure because it must take into account the **actual capability of the hand** to actively exert contact forces.

This introduces two **new ingredients**

- 1) Frictional Forces
- 2) The Hand (!)

Force Closure and Friction

Perhaps, the distinction between FmC and FcC that is most often made in the literature is that **frictional contact forces** are introduced in FcC

Coulomb's Friction Cone Inequality

$$\sigma_{i,f}(\mathbf{p}_i) = \alpha_i \|\mathbf{p}_i\| - \mathbf{p}_i^T \mathbf{n}_i < 0$$

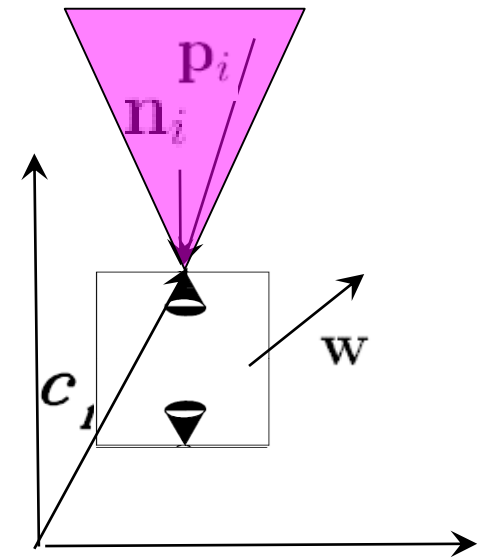
Force/Torque Balance

$$\mathbf{w} = \mathbf{G}\mathbf{p}$$

“Do there exist forces that

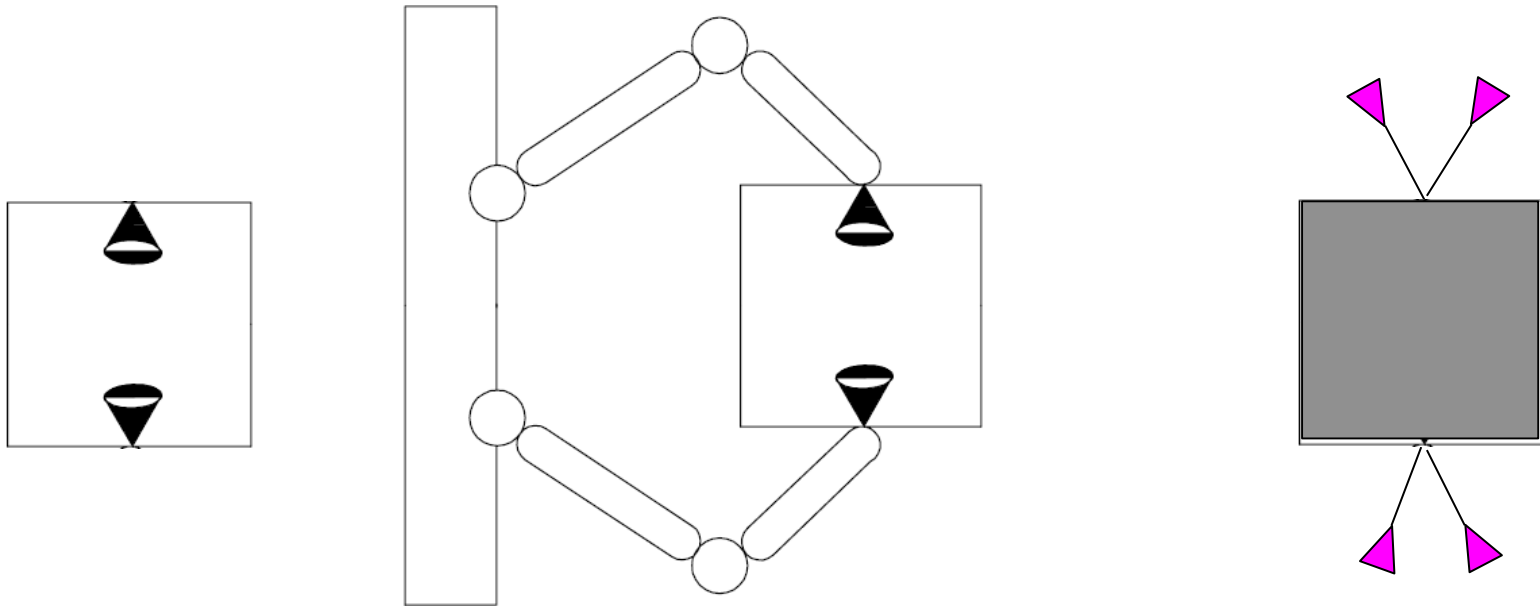
- 1) Balance an arbitrary external load (wrench), and
- 2) Verify all friction constraints?”

...not the right question.



Force Closure and Frictional Form Closure

The frictional nature of contacts is **inessential** for 2D grasps, and of **limited relevance** in general.

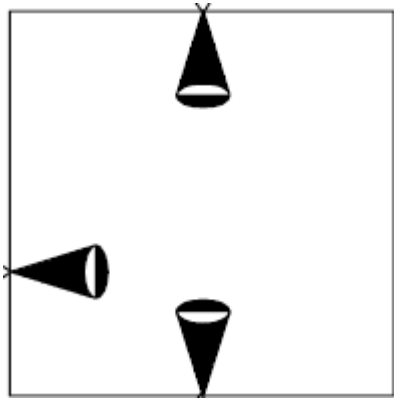


FcC for a frictional 2D grasp with a hand that can arbitrarily control contact forces is equivalent to a FmC problem

In 3D, the same holds exactly for polyhedral friction cones

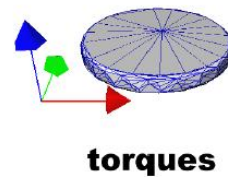
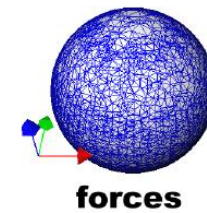
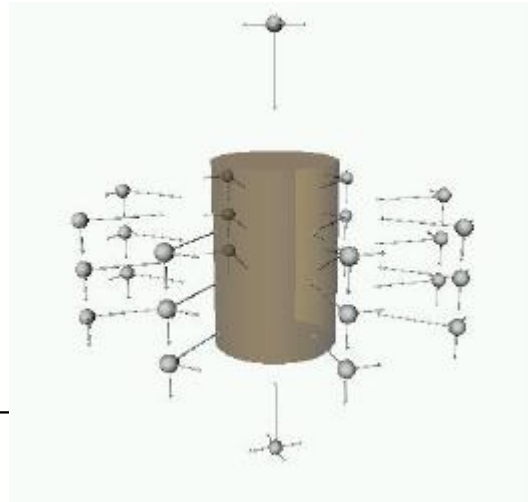
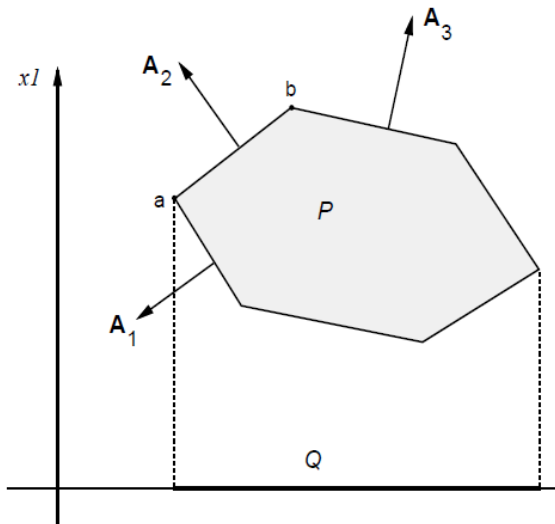
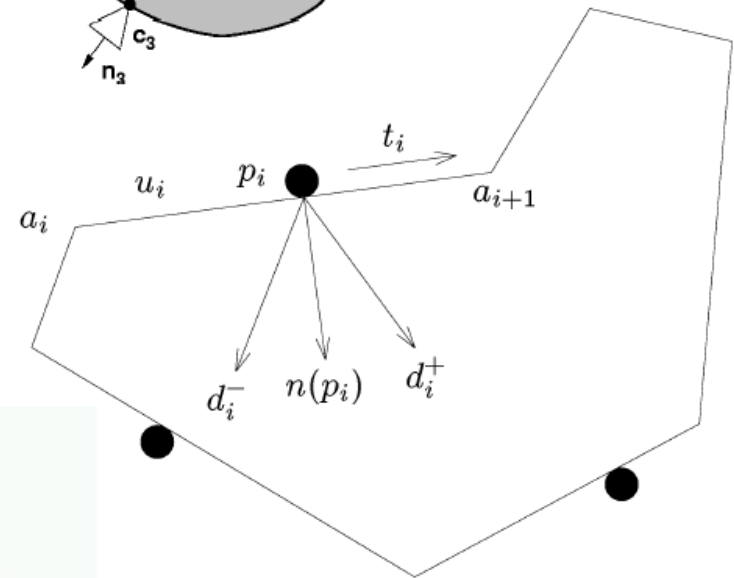
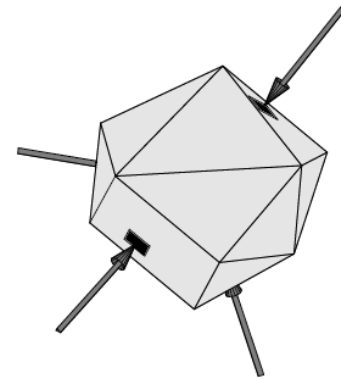
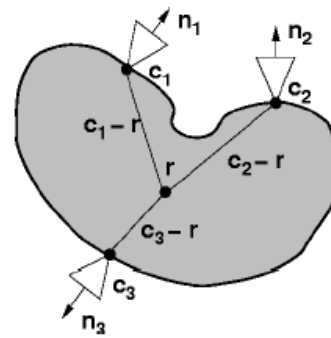
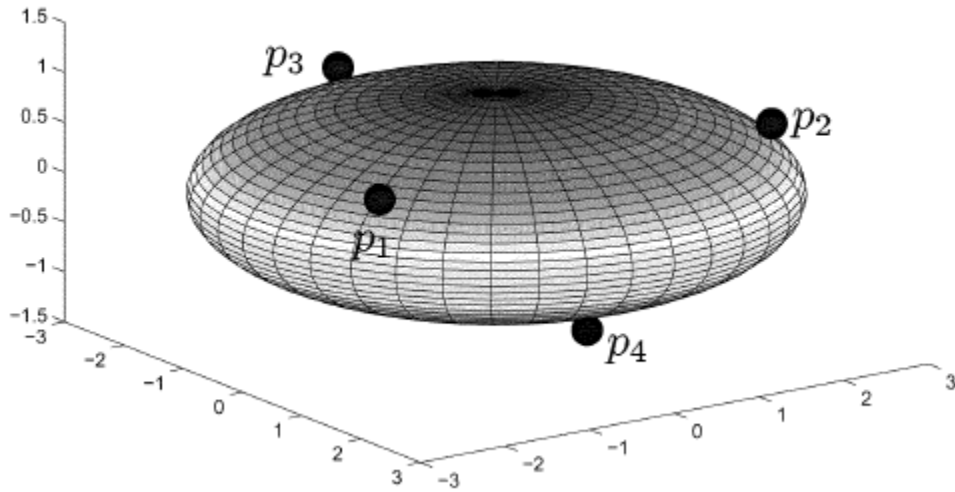
To account for circular cones, *Frictional Form Closure* is probably a better name

Force Closure and the Body



?

Grasping Literature Examples

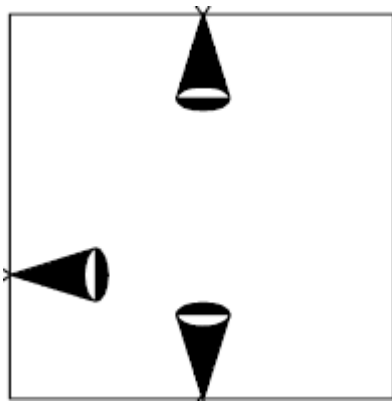


forces

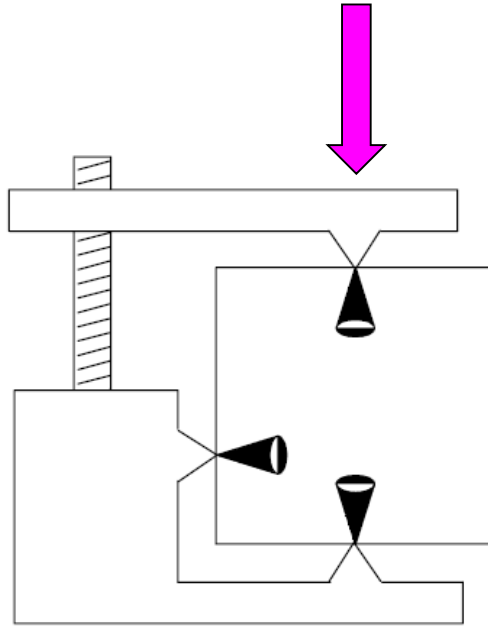
torques

Force Closure and the Body

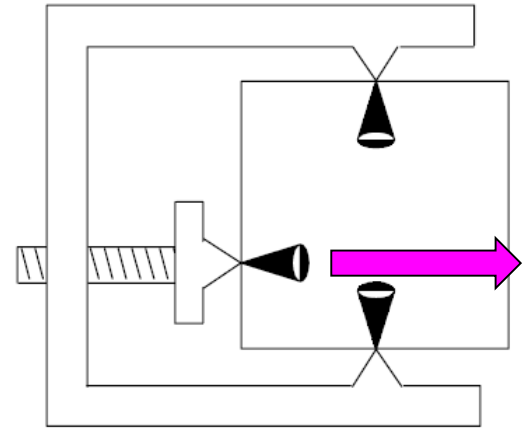
It is the HAND (BODY) that makes the difference!



?



FcC



Not
FcC

Force Closure: Notation and Equations

External load (wrench) \mathbf{w}

Grasp matrix \mathbf{G} (*fat*)

Contact forces \mathbf{p}

$$\mathbf{w} = \mathbf{G}\mathbf{p},$$

Friction Constraints

$$\sigma_{i,f}(\mathbf{p}_i) = \alpha_i \|\mathbf{p}_i\| - \mathbf{p}_i^T \mathbf{n}_i < 0$$

Given \mathbf{w} , which \mathbf{p} ?

$$\mathbf{p} = \mathbf{G}^R \mathbf{w} + \mathbf{A}\mathbf{x},$$

\mathbf{G}^R : Right inverse of \mathbf{G}

\mathbf{A} : a basis of internal forces subspace

By changing \mathbf{x} , squeezing forces are changed: if for every \mathbf{w} it is possible to find \mathbf{x} such that friction constraints are verified, than one has FcC

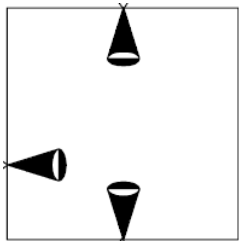
Force Closure and the Grasping Hand

Hand joint torques τ

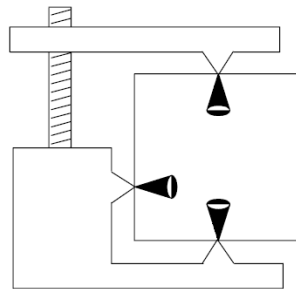
Hand Jacobian \mathbf{J} (*thin*)

$$\tau = \mathbf{J}^T \mathbf{p},$$

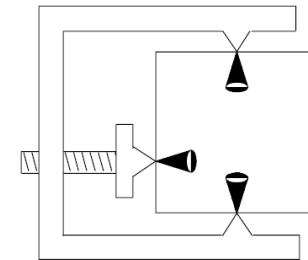
Jacobian not invertible in general \rightarrow can not apply arbitrary contact forces \mathbf{p} !



Grasp
Matrix:
3x6



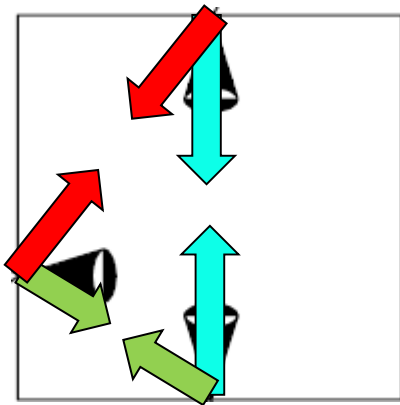
Hand
Jacobian:
6x1



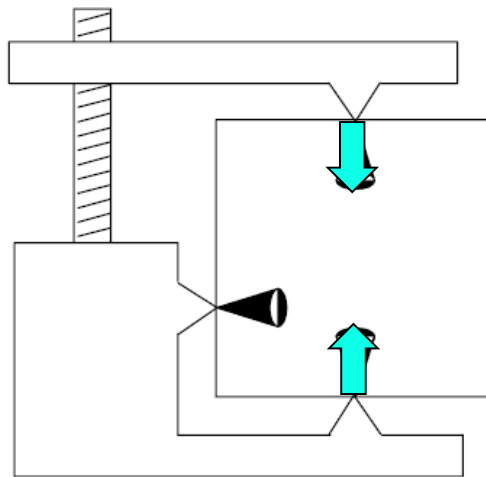
Hand
Jacobian:
6x1

Force Closure and the Grasping Hand

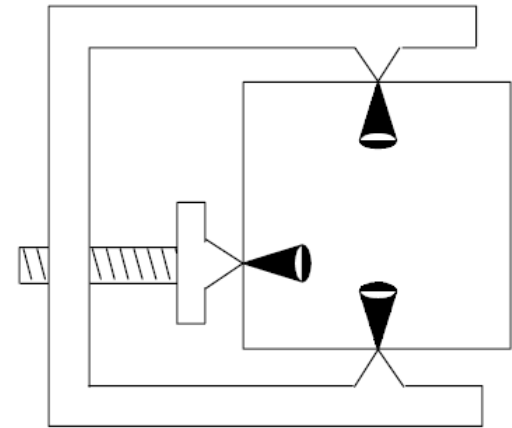
Not all internal forces may be controlled by a real hand!



Internal force subspace is 3-d



Controllable internal force subspace is 1-d



Controllable internal force subspace is 0-d

Rule of thumb: you can never control more internal forces than the number of actuators. But can be less...

Force Closure and the Grasping Hand

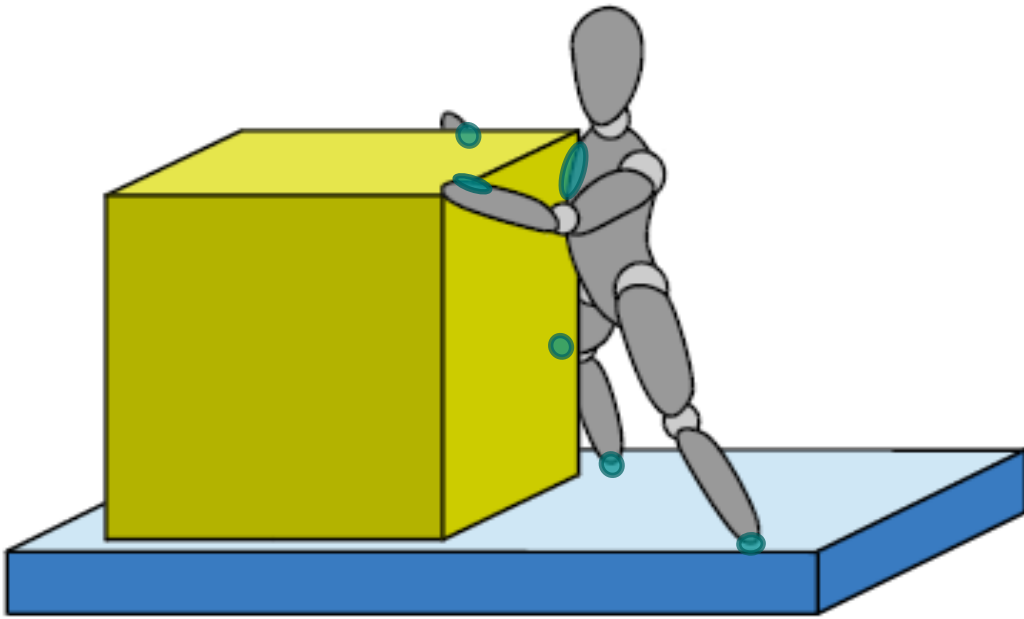


5 contact points
15 contact force comp.s
>15 joints
- ok



5 contact points
15 contact force comp.s
10 joints
- ?

Whole-Body Loco Manipulation



Active Internal Forces

Q: What internal forces at equilibrium are modifiable at will in a given grasp?

$$\mathbf{w} = \mathbf{G}\mathbf{p},$$

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{p},$$

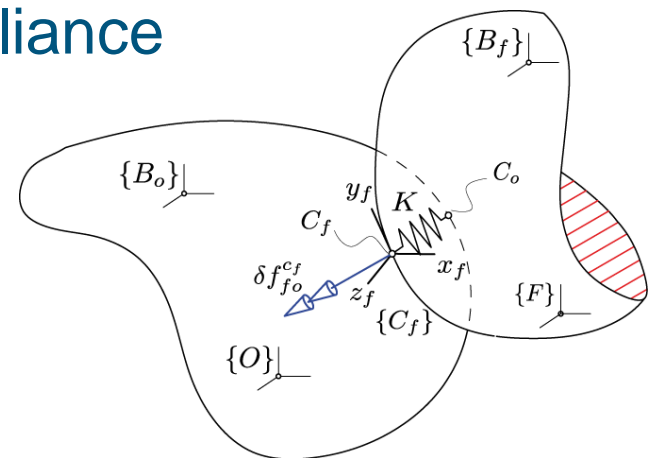
The rigid-body model of grasp is statically indeterminate – no way to determine \mathbf{p} for given \mathbf{w} and $\boldsymbol{\tau}$!

Undergrad mechanics: must introduce congruence and constitutive equations – i.e. compliance

$$\dot{\mathbf{c}}_o = \mathbf{G}^T \dot{\mathbf{u}}$$

$$\dot{\mathbf{c}}_f = \mathbf{J}\dot{\mathbf{q}}$$

$$\mathbf{p} = \mathbf{K}(\mathbf{c}_f - \mathbf{c}_o)$$



Force Distribution in Grasping with Hands

$$\mathbf{w} = \mathbf{G}\mathbf{p},$$

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{p},$$

$$\mathbf{p} = \mathbf{K}(\mathbf{J}\Delta\mathbf{q} - \mathbf{G}^T \Delta\mathbf{u}) + \mathbf{p}_0$$

Well posed

The particular solution $\mathbf{p}_p = \mathbf{G}^R \mathbf{w}$ of the force distribution problem (1) is not unique, since \mathbf{G} in general admits infinitely many right inverses.

However, we expect a unique solution to the following problem:

Force distribution problem.

An object, at equilibrium under an external load \mathbf{w}_0 and contact forces \mathbf{t}_0 , is subject to an additional load \mathbf{w} , while all other parameters (namely \mathbf{t}) are kept constant. **Determine the values of contact forces at the new equilibrium.**

The unique solution, which minimizes the elastic energy and is invariant with coordinate transforms, is

$$\mathbf{p} = \mathbf{G}_K^R \mathbf{w} + \mathbf{p}_0$$
$$\mathbf{G}_K^R := \mathbf{K}\mathbf{G}^T (\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}$$

Internal Forces in Grasping with Hands

- Internal Forces: $\mathbf{p} \in \ker(\mathbf{G})$
- Not all internal forces are active (controllable) acting on the joints

TH: The set of contact forces which can be actively controlled is a linear subspace of $\ker(\mathbf{G})$

$$\text{PLV} \rightarrow \mathbf{Ax} = \mathbf{KJ}\Delta\mathbf{q} - \mathbf{KG}^T \Delta\mathbf{u}$$

$$\text{hence} \quad [\mathbf{A} \quad -\mathbf{KJ} \quad \mathbf{KG}^T] \begin{bmatrix} \mathbf{x} \\ \Delta\mathbf{q} \\ \Delta\mathbf{u} \end{bmatrix} = 0$$

$$\text{and} \quad \mathbf{p}_a = (\mathbf{I} - \mathbf{G}_K^R \mathbf{G}) \mathbf{KJ}\Delta\mathbf{q}$$

$$\mathbf{p}_a = \mathbf{E}\mathbf{y}$$

Preload Forces in Grasping with Hands

- Preload Forces: $\mathbf{p}_p \in \ker(\mathbf{G}) \cap \ker(\mathbf{J}^T)$

The set of passive contact forces is a linear subspace

$$\mathbf{p}_p = \mathbf{Pz}$$

Notice:

- The subspace of active forces changes with compliance
- The subspace of passive forces does not

Consequence: to study grasp with hands, consideration of compliance is **unavoidable**.

In summary: $\mathbf{p} = \mathbf{G}_K^R \mathbf{w} + \mathbf{E}\mathbf{y} + \mathbf{Pz}$

Force Closure

External load (wrench) \mathbf{w}

Grasp matrix \mathbf{G} (*fat*)

Contact forces \mathbf{p}

$$\mathbf{w} = \mathbf{G}\mathbf{p},$$

Friction Constraints

$$\sigma_{i,f}(\mathbf{p}_i) = \alpha_i \|\mathbf{p}_i\| - \mathbf{p}_i^T \mathbf{n}_i < 0$$

Given \mathbf{w} , which \mathbf{p} ?

$$\mathbf{p} = \mathbf{G}_K^R \mathbf{w} + \mathbf{E}\mathbf{y} + \mathbf{P}\mathbf{z}$$

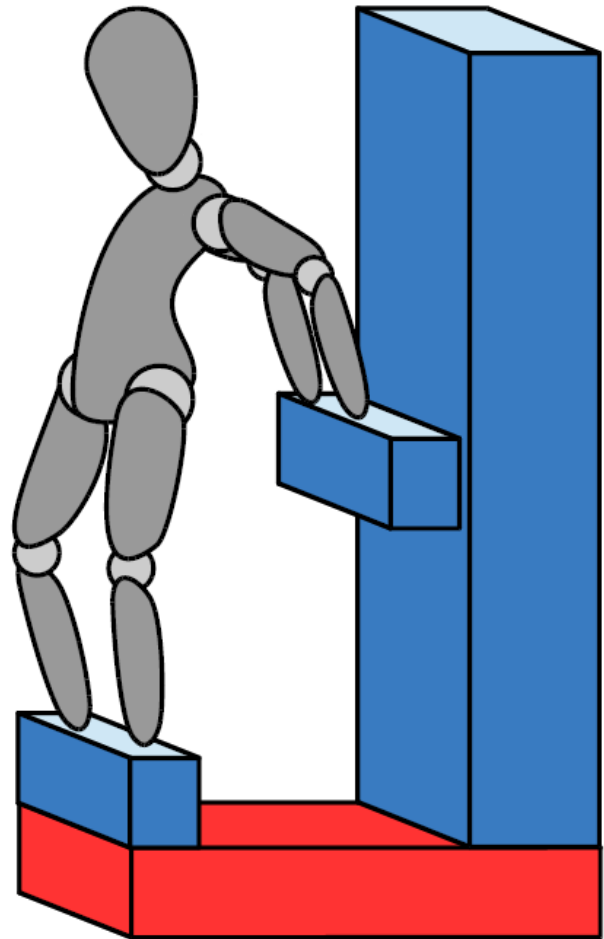
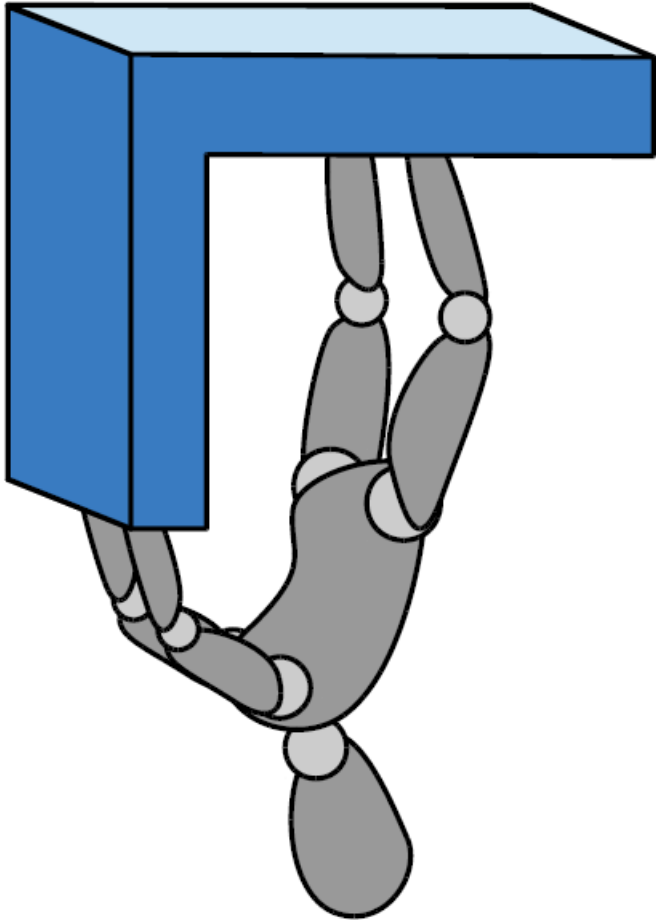
\mathbf{G}_K^R : Right inverse of \mathbf{G}

\mathbf{E} : a basis of internal forces subspace

By changing \mathbf{y} , squeezing forces are changed: if for every \mathbf{w} it is possible to find \mathbf{y} such that friction constraints are verified, than one has FcC

Force Closure: Summary

- The ingredients of FmC are
 - The object, w/h the position and direction of contact constraints
 - The HAND!
- FcC is a **quasi-static** concept:
- FcC test function: $T(c,N,J,K) \rightarrow \{0,1\}$
- Qualitative (yes/no) FcC analysis is a solved problem
- Force optimization is a convex problem
- Contact location is not convex



*Optimal Contact Force Distribution
for
Compliant Humanoid Robots
in
Whole-Body Loco-Manipulation Tasks*

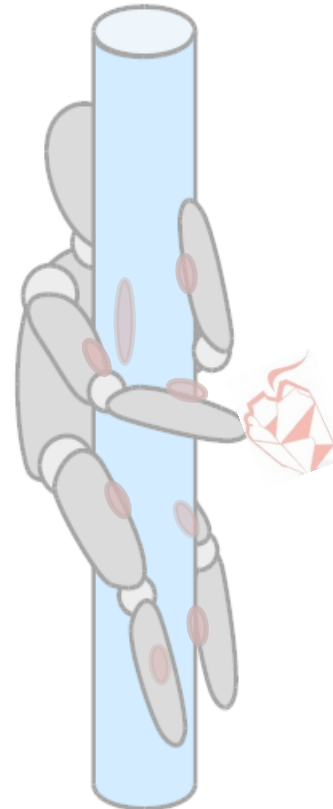
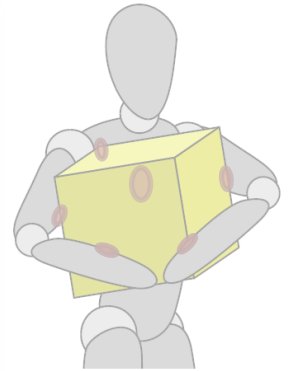
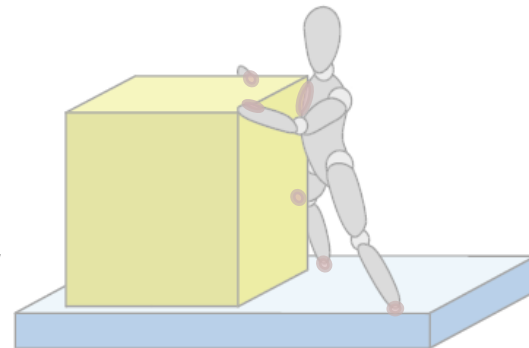
Edoardo Farnioli, Marco Gabiccini, Antonio Bicchi



University of Pisa



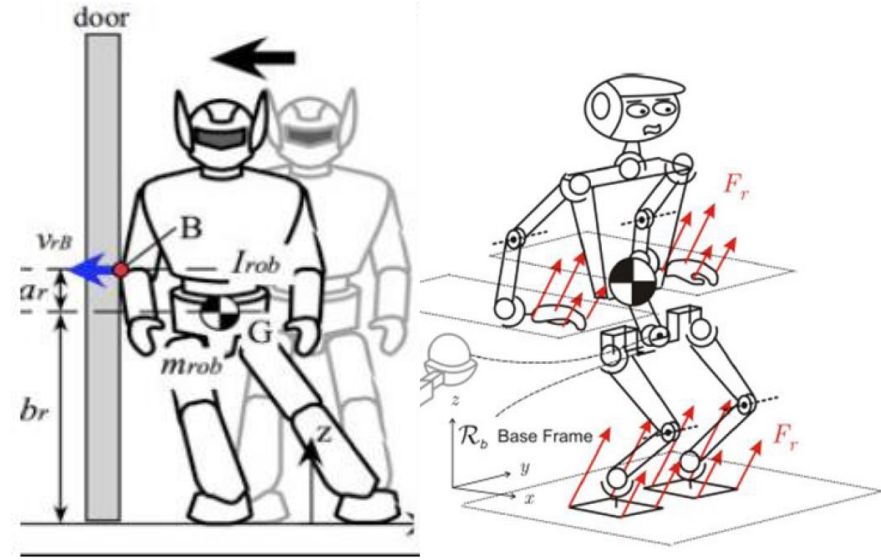
Italian Institute of
Technology



Whole-Body Loco-Manipulation: Reference Scenario

Often, full contact force controllability is assumed, by virtue of the high number of DoF of the Humanoid Robots

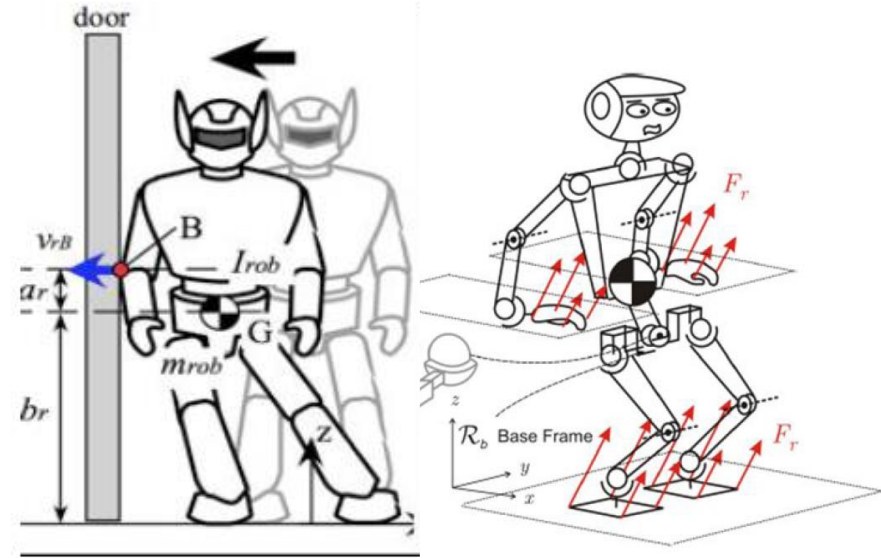
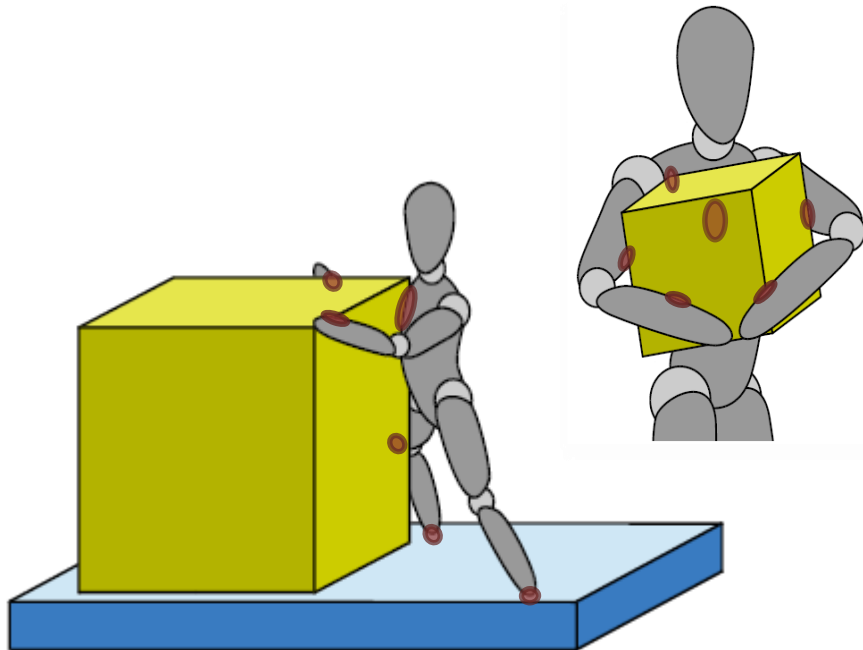
Is it true also for whole-body interactions?



Whole-Body Loco-Manipulation: Reference Scenario

Often, full contact force controllability is assumed, by virtue of the high number of DoF of the Humanoid Robots

Is it true also for whole-body interactions?

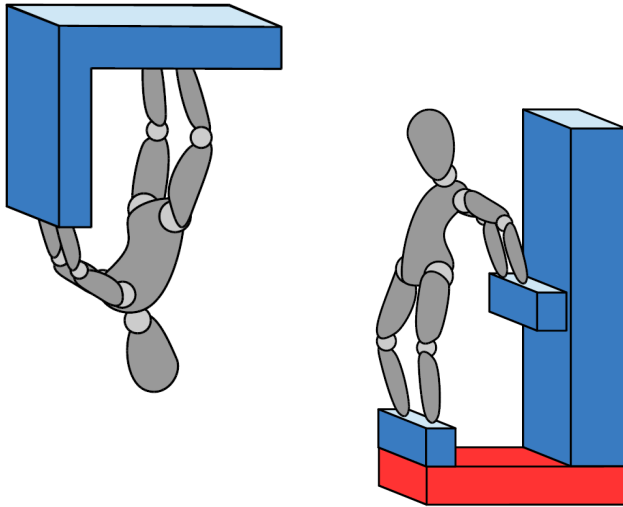


In *Whole-Body Loco-Manipulation* the robot-environment interaction can occur also on the internal limbs

Problems:

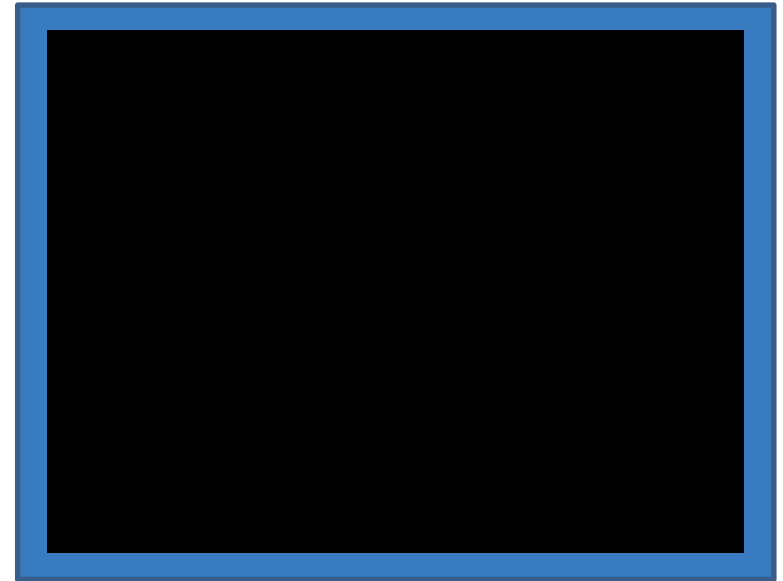
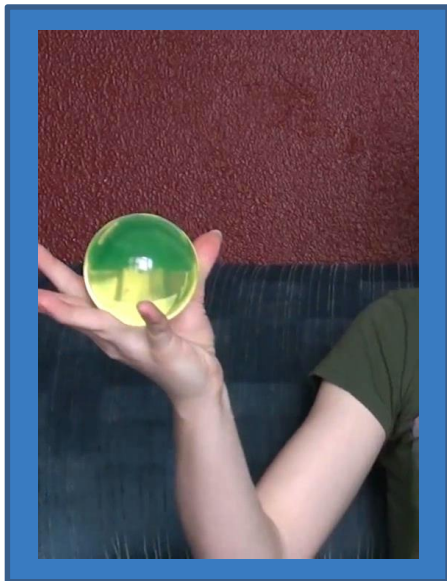
- kinematic chains are locally defective
- less joints than force components to be controlled
- cannot control contact forces arbitrarily

Grasping Vs. Loco-Manipulation (1/2)



It is not new the observations that the two systems are similar. Probably the similarities can be reconduced to relativity Galileo's principle

Can we use the same analysis tools for analyzing both the systems?



Whole-Body Loco-Manipulation: Quasi-Static Model

Twists of the contact frames

$$\xi_{ac}^c = \begin{bmatrix} {}^c J_v & {}^c J \end{bmatrix} \begin{bmatrix} \dot{q}_v \\ \dot{q} \end{bmatrix}$$

where

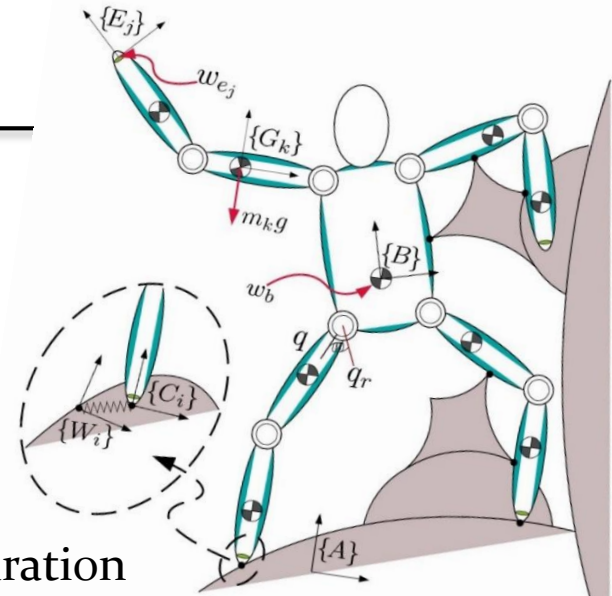
$u := q_v$ parametrizing the floating base configuration

$S^T := {}^c J_v$ defining the **Stance Matrix**, mapping the floating frame into end-effector displacements

by kineto-static duality

$\delta w = S \delta f_c$ equilibrium of the floating base

$\delta \tau = {}^c J^T \delta f_c$ equilibrium of the (real) joints



NOTATION

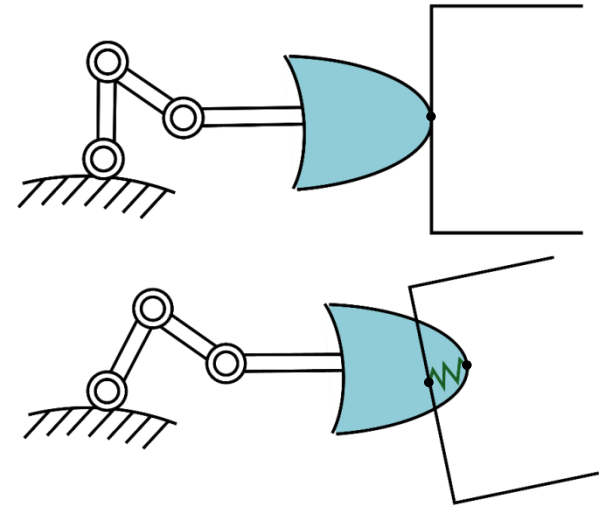
- q robot joint parameters
- u virtual kin. chain joint parameters
- τ robot joint torques
- w external wrench = VKC joint torques
- f_c contact forces

Whole-Body Loco-Manipulation: Quasi-Static Model

Constitutive equation of the contact forces

$$\delta f_c = K_c(J\delta q + S^T \delta u)$$

penalty formulation: the contact forces born in
case of robot/environment compenetration



NOTATION

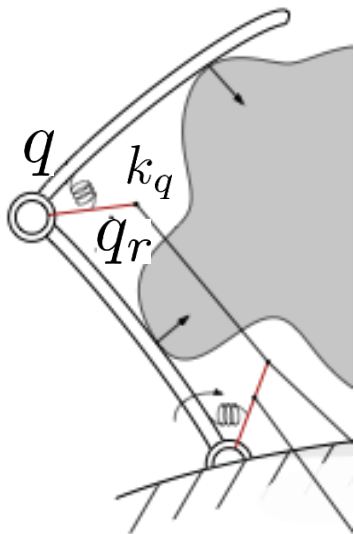
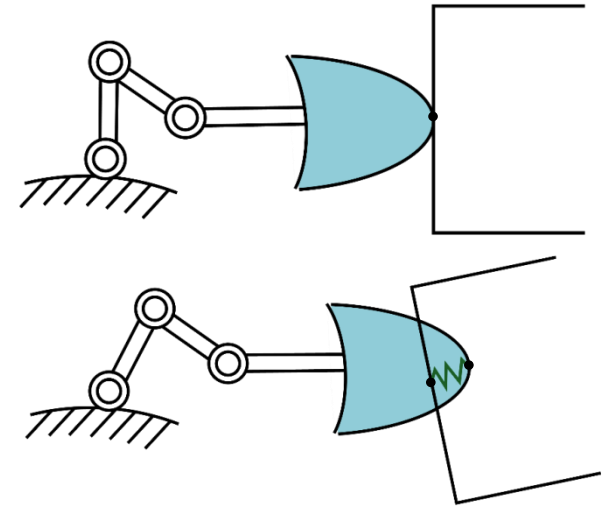
q	robot joint parameters
u	virtual kin. chain joint parameters
τ	robot joint torques
w	external wrench = VKC joint torques
f_c	contact forces

Whole-Body Loco-Manipulation: Quasi-Static Model

Constitutive equation of the contact forces

$$\delta f_c = K_c (J \delta q + S^T \delta u)$$

penalty formulation: the contact forces born in case of robot/environment compenetration



Constitutive equation of the elatic joints

$$\delta \tau = K_q (\delta q_r - \delta q)$$

the joint torque is described by the miasmach between the real joint configuration and its reference value

NOTATION

q	robot joint parameters
u	virtual kin. chain joint parameters
τ	robot joint torques
w	external wrench = VKC joint torques
f_c	contact forces

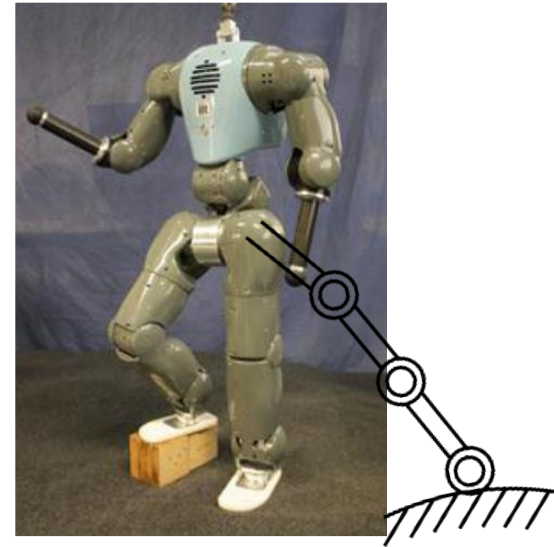
Quasi-Static Loco-Manipulation Equation

Grouping together all the equations we obtain

$$\begin{bmatrix} I_f & 0 & -K_c {}^c S^T & -K_c {}^c J & 0 & 0 \\ -{}^c J^T & I_\tau & -U_j & -Q_j & 0 & 0 \\ -{}^c S & 0 & -U_s & -Q_s & I_w & 0 \\ 0 & I_\tau & 0 & K_q & 0 & -K_q \end{bmatrix} \begin{bmatrix} \delta f_c \\ \delta \tau \\ \delta u \\ \delta q \\ \delta w \\ \delta q_r \end{bmatrix} = 0$$

where we introduced $U_s = \frac{\partial {}^c S f_c}{\partial u}$, $U_j = \frac{\partial {}^c J^T f_c}{\partial u}$, $Q_s = \frac{\partial {}^c S f_c}{\partial q}$, $Q_j = \frac{\partial {}^c J^T f_c}{\partial u}$,

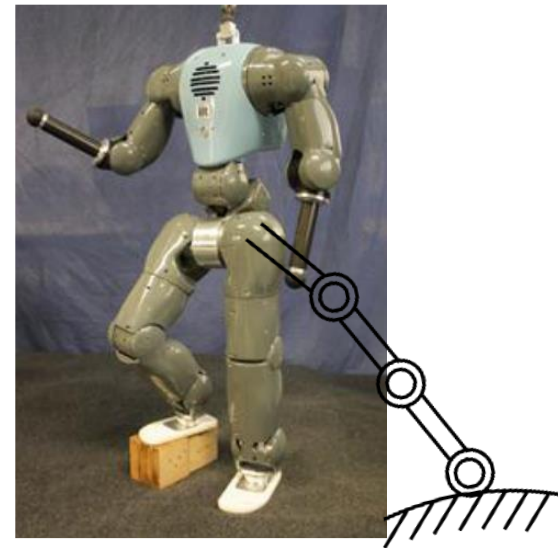
for properly consider contact force preload.



Quasi-Static Loco-Manipulation Equation

Grouping together all the equations we obtain

$$\begin{bmatrix} I_f & 0 & -K_c^c S^T & -K_c^c J & 0 & 0 \\ -{}^c J^T & I_\tau & -U_j & -Q_j & 0 & 0 \\ -{}^c S & 0 & -U_s & -Q_s & I_w & 0 \\ 0 & I_\tau & 0 & K_q & 0 & -K_q \end{bmatrix} \begin{bmatrix} \delta f_c \\ \delta \tau \\ \delta u \\ \delta q \\ \delta w \\ \delta q_r \end{bmatrix} = 0$$

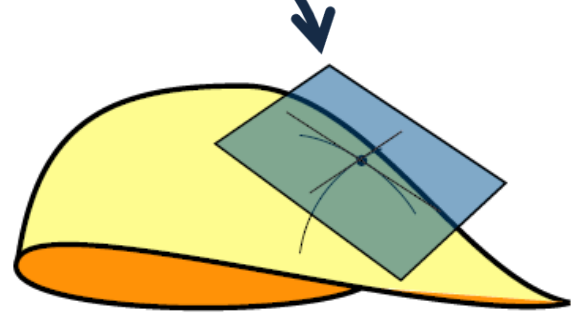


where we introduced $U_s = \frac{\partial {}^c S f_c}{\partial u}$, $U_j = \frac{\partial {}^c J^T f_c}{\partial u}$, $Q_s = \frac{\partial {}^c S f_c}{\partial q}$, $Q_j = \frac{\partial {}^c J^T f_c}{\partial u}$,

for properly consider contact force preload.

Geometrical Interpretation:

the **Quasi-Static Loco-Manipulation Equation** is the analytical description of the hyperplane tangent to the equilibrium space of the system.



Can this equation tell us something interesting on the system?

Canonical Form of the Quasi-Static Loco-Manipulation Equation: Computation

$$\underbrace{\begin{bmatrix} I_f & 0 & -K_c^c S^T & -K_c^c J \\ -{}^c J^T & I_\tau & -U_j & -Q_j \\ -{}^c S & 0 & -U_s & -Q_s \\ 0 & I_\tau & 0 & K_q \end{bmatrix}}_{\text{dependent variables}} \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ I_w & 0 \\ 0 & -K_q \end{bmatrix}}_{\text{independent variables}} \underbrace{\begin{bmatrix} \delta f_c \\ \delta \tau \\ \delta u \\ \delta q \\ \delta w \\ \delta q_r \end{bmatrix}}_{\text{independent variables}} = 0$$

$$\Rightarrow \begin{bmatrix} \Phi_d & \Phi_i \end{bmatrix} \begin{bmatrix} \delta \varphi_d \\ \delta \varphi_i \end{bmatrix} = 0$$

$$\Phi_d^{-1} \begin{bmatrix} \Phi_d & \Phi_i \end{bmatrix} \begin{bmatrix} \delta \varphi_d \\ \delta \varphi_i \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} I_{\varphi_d} & \Phi_i^* \end{bmatrix} \begin{bmatrix} \delta \varphi_d \\ \delta \varphi_i \end{bmatrix} = 0$$

Canonical Form of the Quasi-Static Loco-Manipulation Equation (FLME)

$$\begin{bmatrix} I_f & 0 & 0 & 0 & W_f & R_f \\ 0 & I_\tau & 0 & 0 & W_\tau & R_\tau \\ 0 & 0 & I_u & 0 & W_u & R_u \\ 0 & 0 & 0 & I_q & W_q & R_q \end{bmatrix} \begin{bmatrix} \delta f_c \\ \delta \tau \\ \delta u \\ \delta q \\ \delta w \\ \delta q_r \end{bmatrix} = 0$$

Controllable Contact Force Variations

from the first eq. $\delta f_c = -R_f \delta q_r - W_f \delta w$

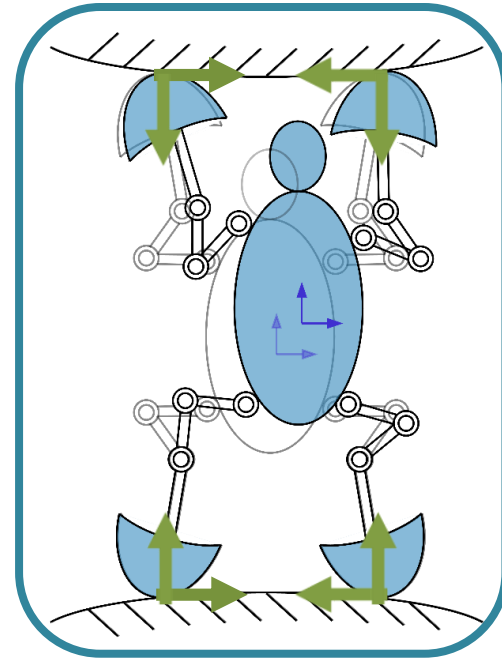
In other words, if no external wrench is acting on the robot, the contact force variation consequent to a joint configuration variation is

$$\delta f_c = -R_f \delta q_r$$

or also

$$\delta f_c = -R_f \delta q_r = E y$$

where E is a basis for the Controllable Contact Forces



How can we properly consider contact force limits? (e.g. friction cone)

Contact Force Constraints: Metric

friction cone

$$\sigma_{i,\text{frict}} = \alpha_i \|f_{c_i}\| - f_{c_i}^T n_i \leq 0$$

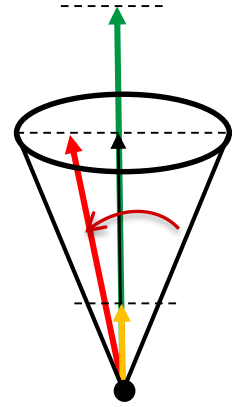
minimum force

$$\sigma_{i,\text{min}} = f_{\text{min}_i} - f_{c_i}^T n_i \leq 0$$

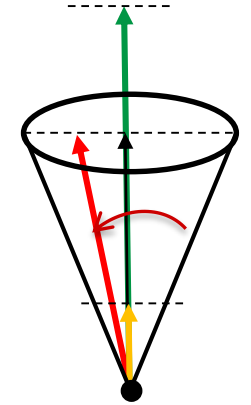
maximum force

$$\sigma_{i,\text{max}} = -f_{\text{max}_i} + f_{c_i}^T n_i \leq 0$$

$$\sigma_{i,j} \leq 0$$

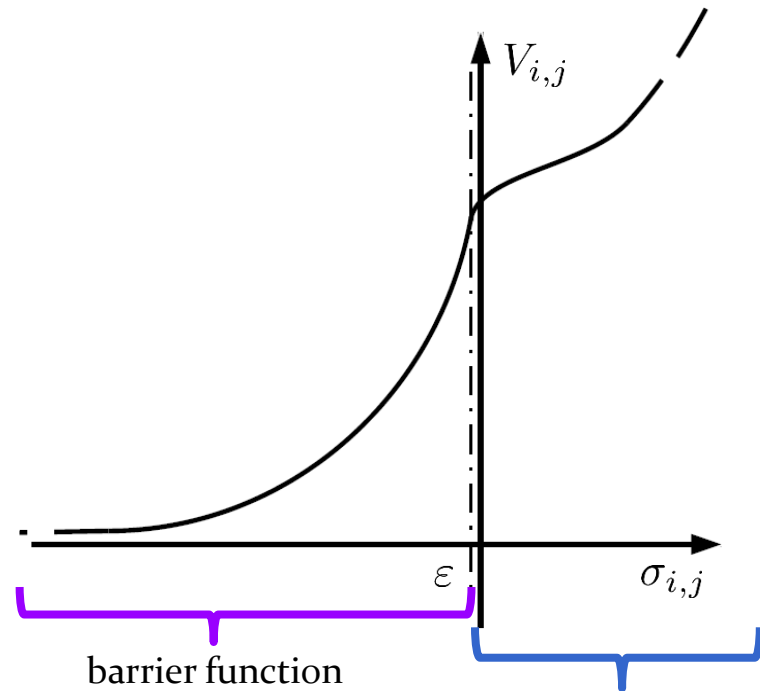


Contact Force Constraints: Metric

<u>friction cone</u>	$\sigma_{i,\text{frict}} = \alpha_i \ f_{c_i}\ - f_{c_i}^T n_i \leq 0$	} $\sigma_{i,j} \leq 0$ }	
<u>minimum force</u>	$\sigma_{i,\text{min}} = f_{\text{min}_i} - f_{c_i}^T n_i \leq 0$		
<u>maximum force</u>	$\sigma_{i,\text{max}} = -f_{\text{max}_i} + f_{c_i}^T n_i \leq 0$		

$$V_{i,j} = \begin{cases} \frac{(2\sigma_{i,j}^2)^{-1}}{a\sigma_{i,j}^2 + b\sigma_{i,j} + c} & \text{if } \sigma_{i,j} < \varepsilon \\ a\sigma_{i,j}^2 + b\sigma_{i,j} + c & \text{otherwise} \end{cases}$$

$$V = \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{T}} V_{i,j}$$



Contact Force Optimization is a Convex Problem!

Theorem: Remembering that $\delta f_c = -R_f \delta q = Ey$

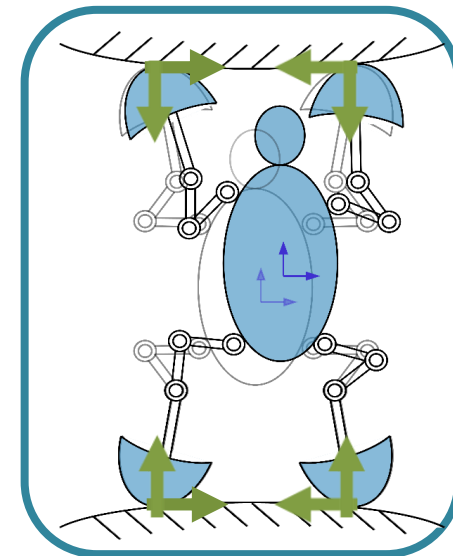
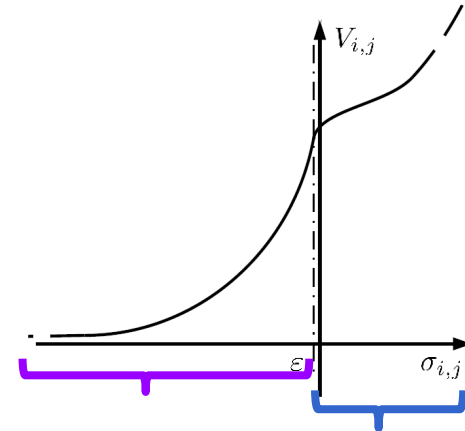
the contact forces can be expressed as

$$f_c = f_0 + \delta f = f_0 + Ey$$

then we can prove that the function

$$V(y) = \sum V_{i,j} \quad \text{is convex,}$$

and $\sigma_{i,j} \leq 0$; $\forall i, j$ describes a **convex set**



Contact Force Optimization is a Convex Problem!

Theorem: Remembering that $\delta f_c = -R_f \delta q = Ey$

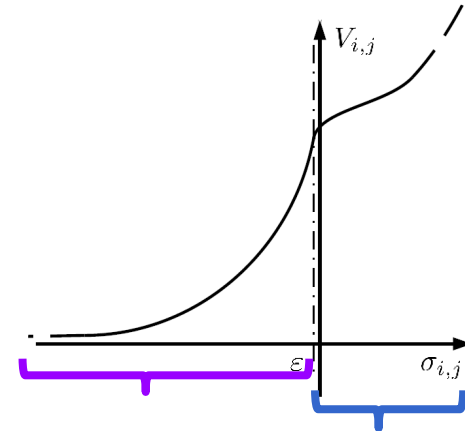
the contact forces can be expressed as

$$f_c = f_0 + \delta f = f_0 + Ey$$

then we can prove that the function

$$V(y) = \sum V_{i,j} \text{ is convex,}$$

and $\sigma_{i,j} \leq 0; \forall i,j$ describes a **convex set**

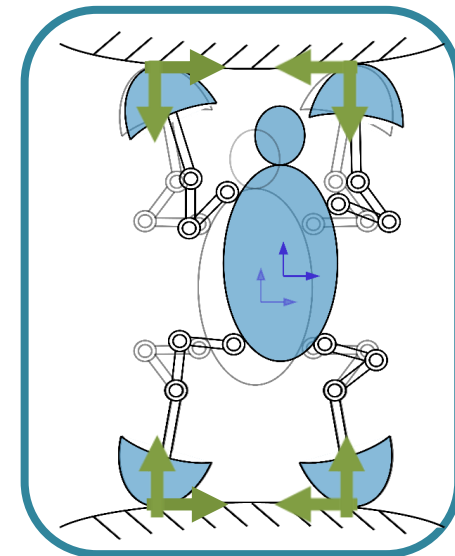


Once the minimum of V is found, the optimal contact force variation follows

$$\hat{y} = \arg \min V(y) \implies \widehat{\delta f_c} = E\hat{y}$$

From previous results it follows that the optimal joint variation as

$$\widehat{\delta q_r} = -R_f^\dagger \widehat{\delta f_c} + \Gamma_{R_f} z$$



Numerical Example (1/2)

Pushing Without Slipping

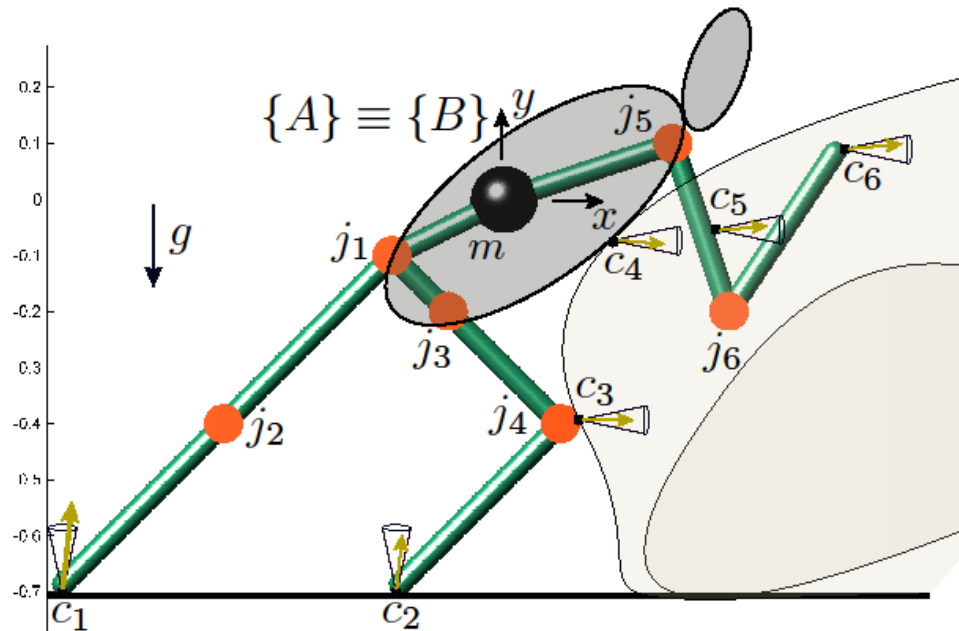
In the case in which the feet are hooked to the ground (no contact limits), the FOP provides

$$\text{Total pushing force} \quad f_{x_tot} = 666.6 \text{ N}$$

$$\text{with contact forces on feet} \quad f_{c_1} = [-839.3, 1055]^T \text{ N}$$

$$f_{c_2} = [391.6, -20.9]^T \text{ N}$$

exploiting bilateral interactions with the environment.



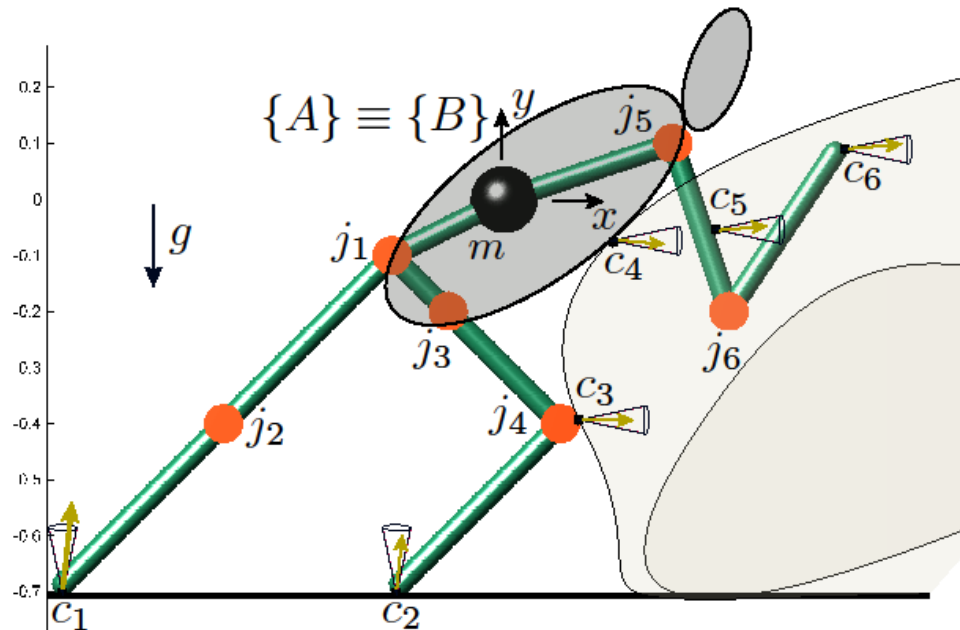
Numerical Example (1/2)

Pushing Without Slipping

In the case in which the feet are hooked to the ground (no contact limits), the FOP provides

Total pushing force	$f_{x_tot} = 666.6 \text{ N}$
with contact forces on feet	$f_{c_1} = [-839.3, 1055]^T \text{ N}$
	$f_{c_2} = [391.6, -20.9]^T \text{ N}$

exploiting bilateral interactions with the environment.



Instead, if unilateral constraints are imposed on the feet, such that

$$f_{max} = 500 \text{ N} \quad f_{min} = 0 \text{ N}$$

the total pushing force becomes

$$f_{x_tot} = 528.6 \text{ N}$$

meeting all the contact constraints, thus without slipping!

Numerical Example (2/2)

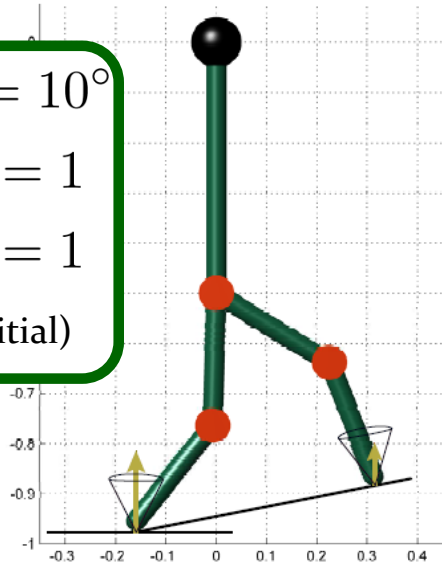
Balancing on a Slippery Slope

$$\alpha = 10^\circ$$

$$\mu_1 = 1$$

$$\mu_2 = 1$$

(initial)

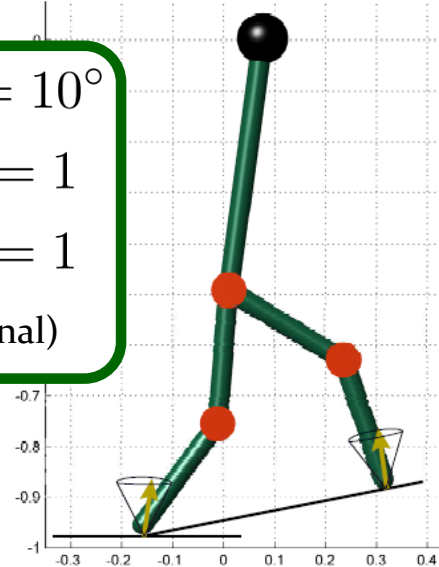


$$\alpha = 10^\circ$$

$$\mu_1 = 1$$

$$\mu_2 = 1$$

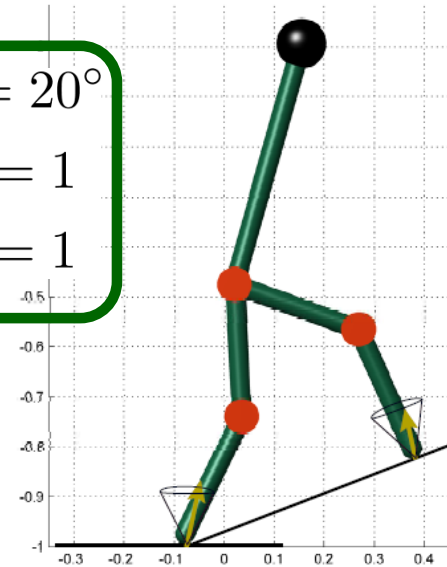
(final)



$$\alpha = 20^\circ$$

$$\mu_1 = 1$$

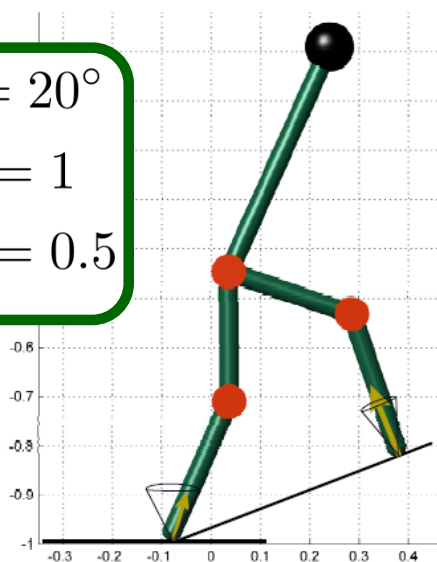
$$\mu_2 = 1$$



$$\alpha = 20^\circ$$

$$\mu_1 = 1$$

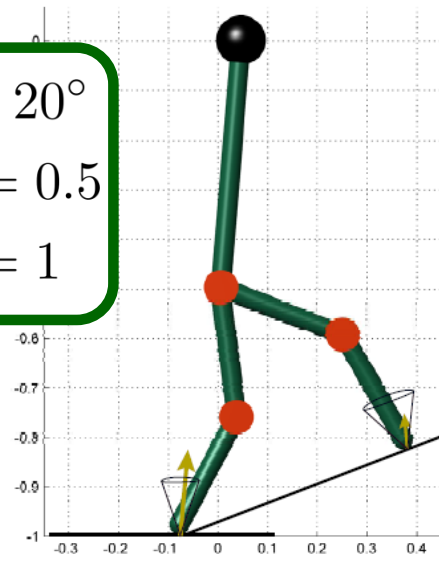
$$\mu_2 = 0.5$$



$$\alpha = 20^\circ$$

$$\mu_1 = 0.5$$

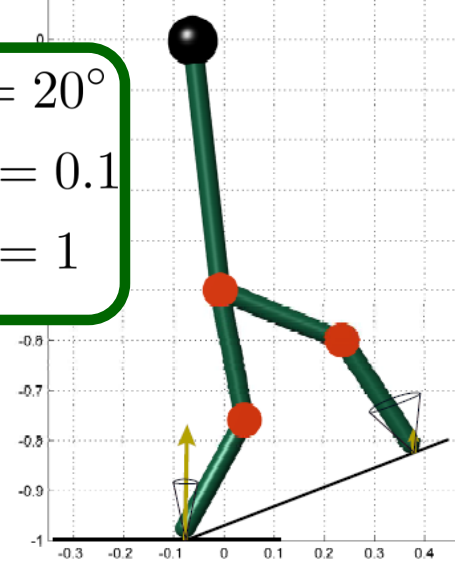
$$\mu_2 = 1$$



$$\alpha = 20^\circ$$

$$\mu_1 = 0.1$$

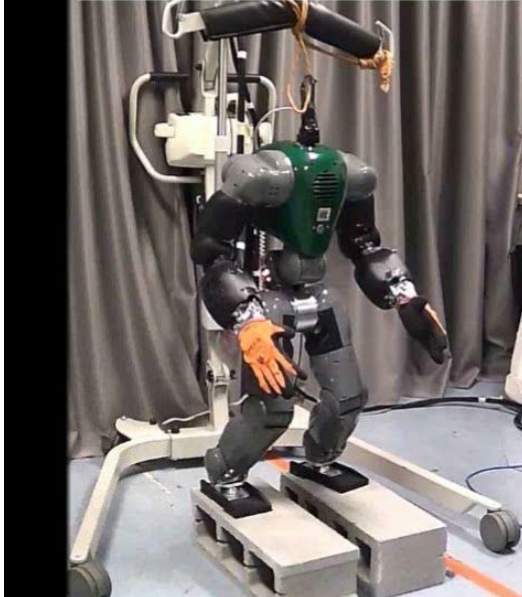
$$\mu_2 = 1$$



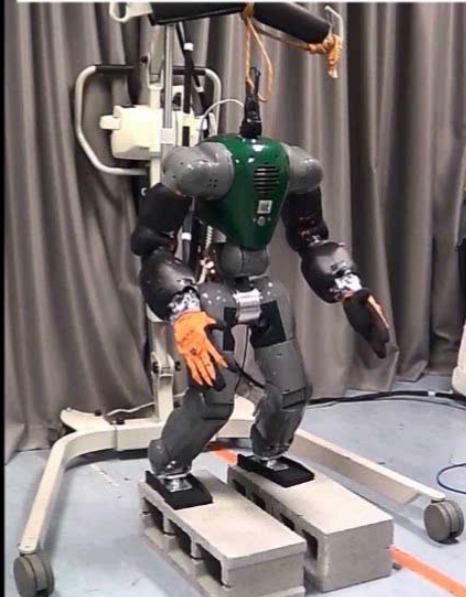
Experimental Test: Balancing on Flat Terrain

Robot Posture Adapts to Friction Condition

Building the function $V(y)$
with $\mu_l = 1, \mu_r = 0.3$



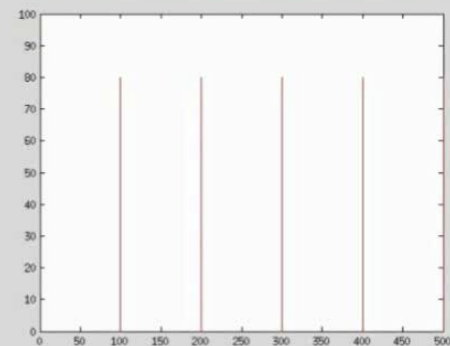
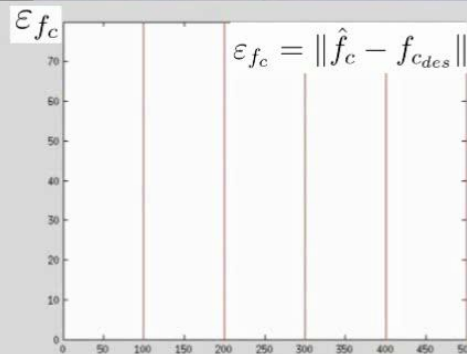
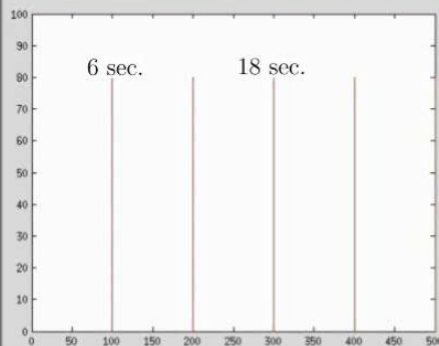
Building the function $V(y)$
with $\mu_l = 1, \mu_r = 1$



Building the function $V(y)$
with $\mu_l = 0.3, \mu_r = 1$

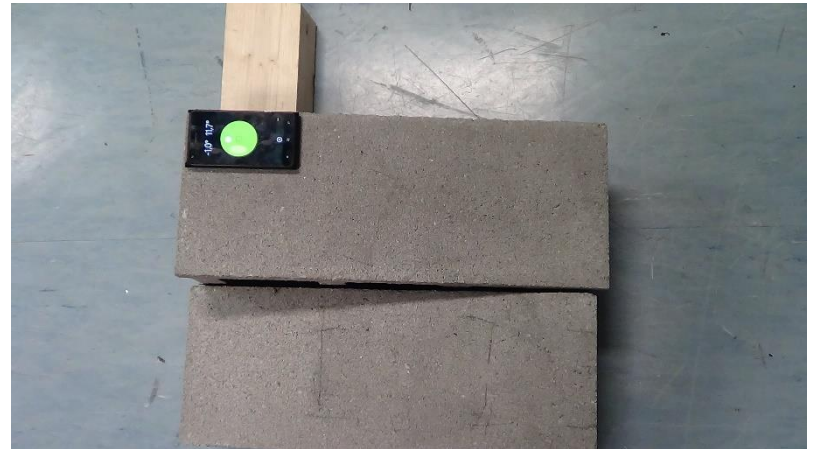
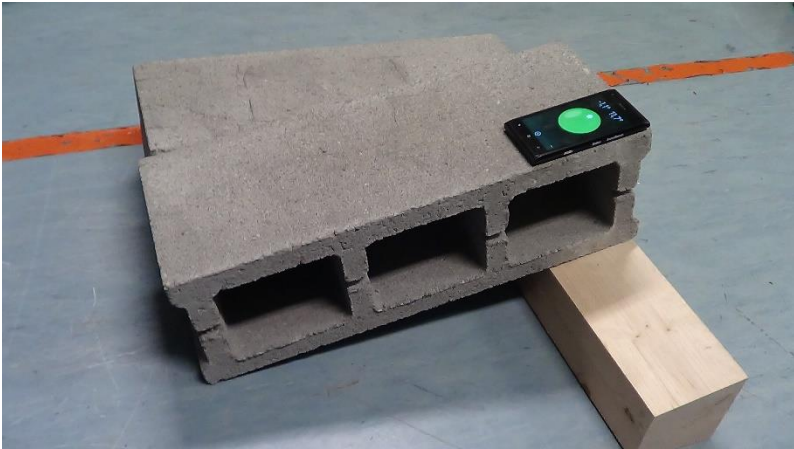


3X



Experimental Test: Balancing on Half-Uneven Terrain Environment

Half-Uneven Terrain: left foot on flat terrain, right foot on slope of $\simeq 10^\circ$

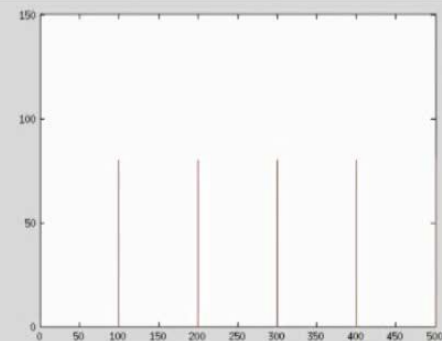
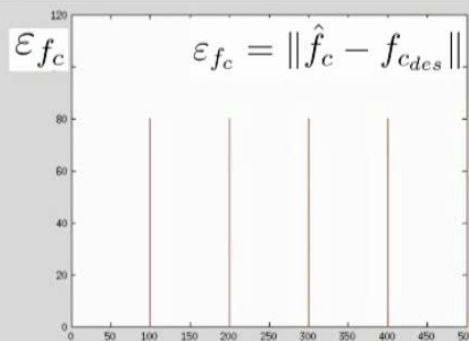
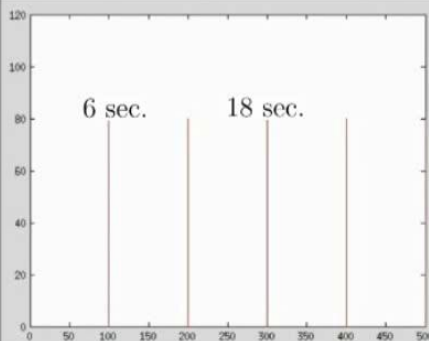


Experimental Test: Balancing on Half-Uneven Terrain

Robot Posture Adapts to Slope and Friction Condition

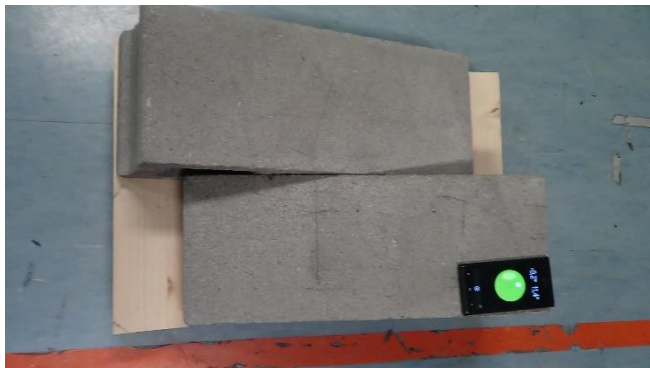


3X

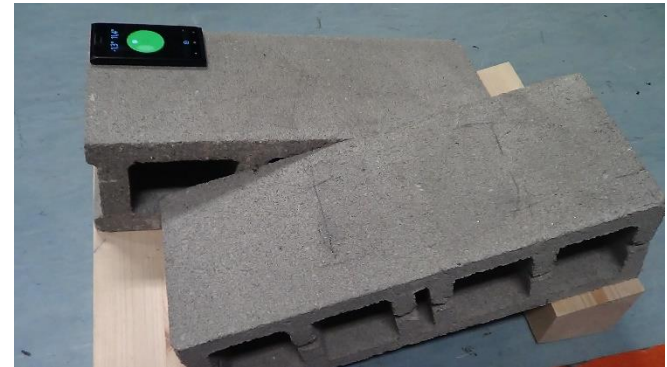


Experimental Test: Balancing on Uneven Terrain (1/2) Environment

Left foot on a slope of $\simeq -10^\circ$

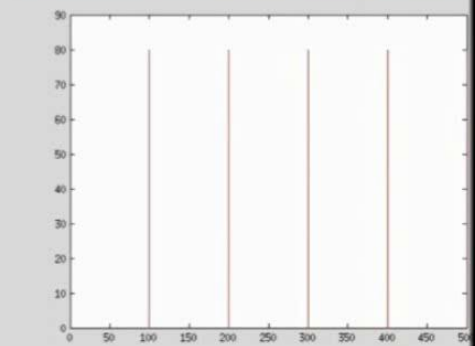
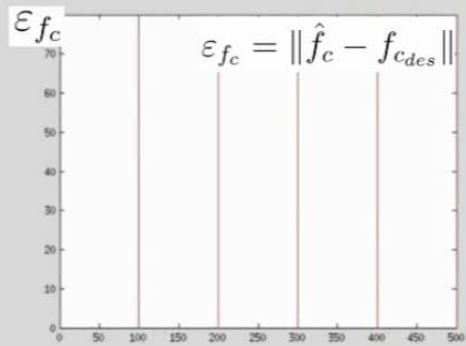
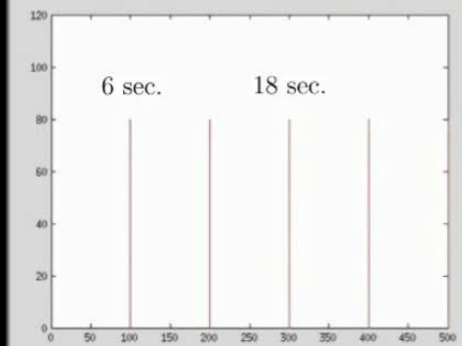


Right foot on slope of $\simeq 10^\circ$



Experimental Test: Balancing on Uneven Terrain (1/2)

Robot Posture Adapts to Slope and Friction Condition

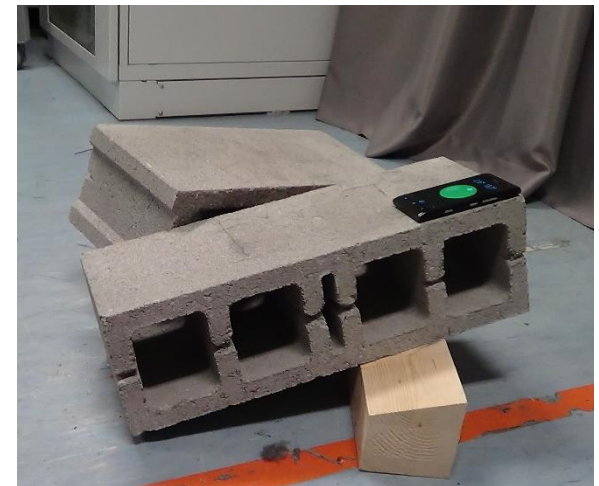


Experimental Test: Balancing on Uneven Terrain (2/2) Environment

Left Foot



Right Foot



Experimental Test: Balancing on Uneven Terrain (2/2)

Robot Posture Adapts to Slope and Friction Condition

Building the function $V(y)$
with $\mu_l = 1, \mu_r = 0.4$



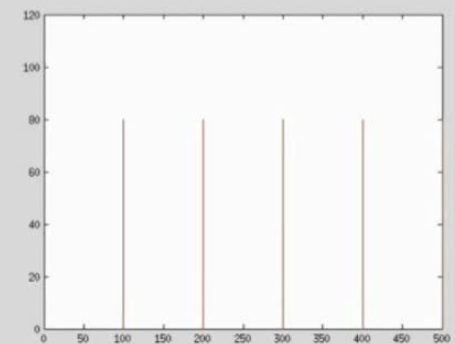
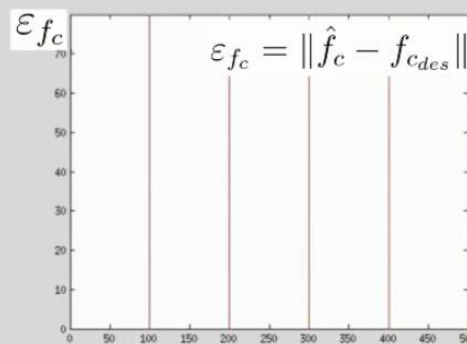
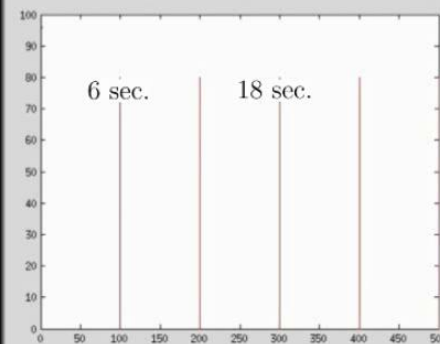
Building the function $V(y)$
with $\mu_l = 1, \mu_r = 1$



Building the function $V(y)$
with $\mu_l = 0.4, \mu_r = 1$



3X



*Optimal Contact Force Distribution
for
Compliant Humanoid Robots
in
Whole-Body Loco-Manipulation Tasks*

Edoardo Farnioli, Marco Gabiccini, Antonio Bicchi



University of Pisa



Italian Institute of
Technology

